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# Tensions in Cosmology: A signal of Modified Gravity?

*Emmanuel N. Saridakis*

*National Observatory of Athens*

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*Cosmoverse@Lisbon*



- The **history** of **Astronomy, Cosmology** and **Gravity** is a **history of tensions** between **theoretical predictions** and **observations**
- **Astrophysical cosmology** has become a **precision science** with an **incredibly huge amount of data**
- **New Tensions appear.**  
**Are we approaching New Physics?**

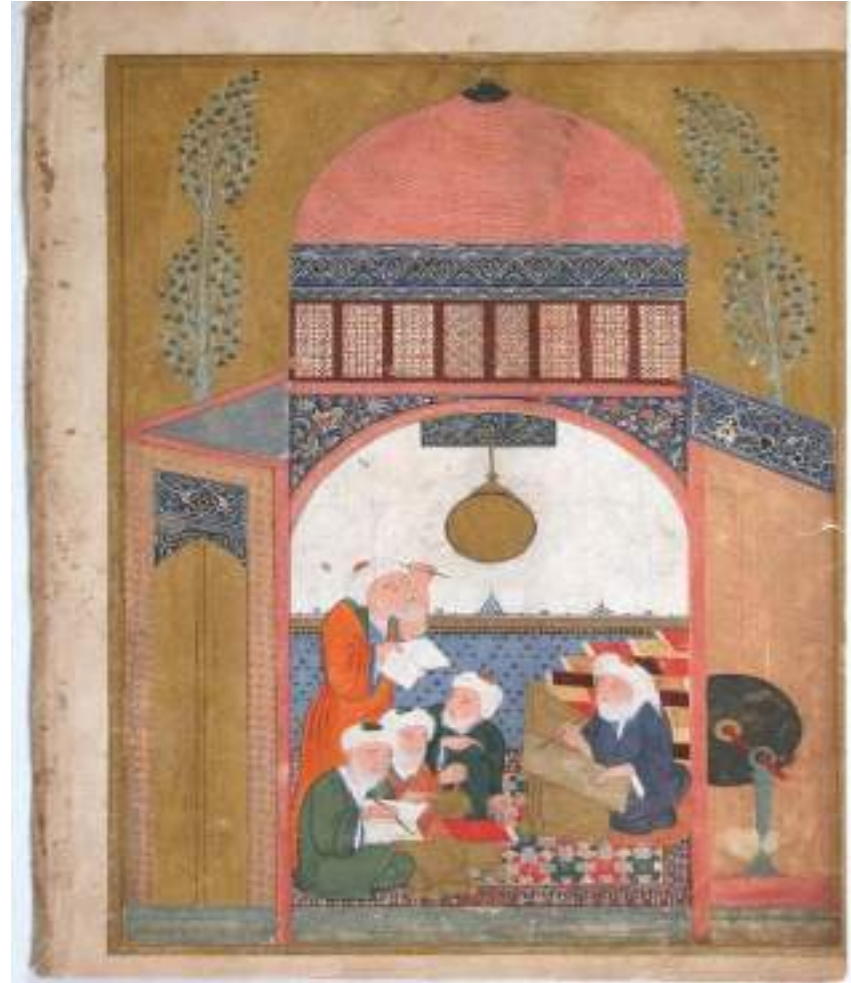
# Aristotle - 350 BC

- According to Aristotle heavier bodies fall faster.
- Bodies fall in order to come back to their "initial state".



# Maragha Observations

- Observations in Maragha in 11<sup>th</sup> century, started putting into doubt Earth's non-motion, however not geocentrism.



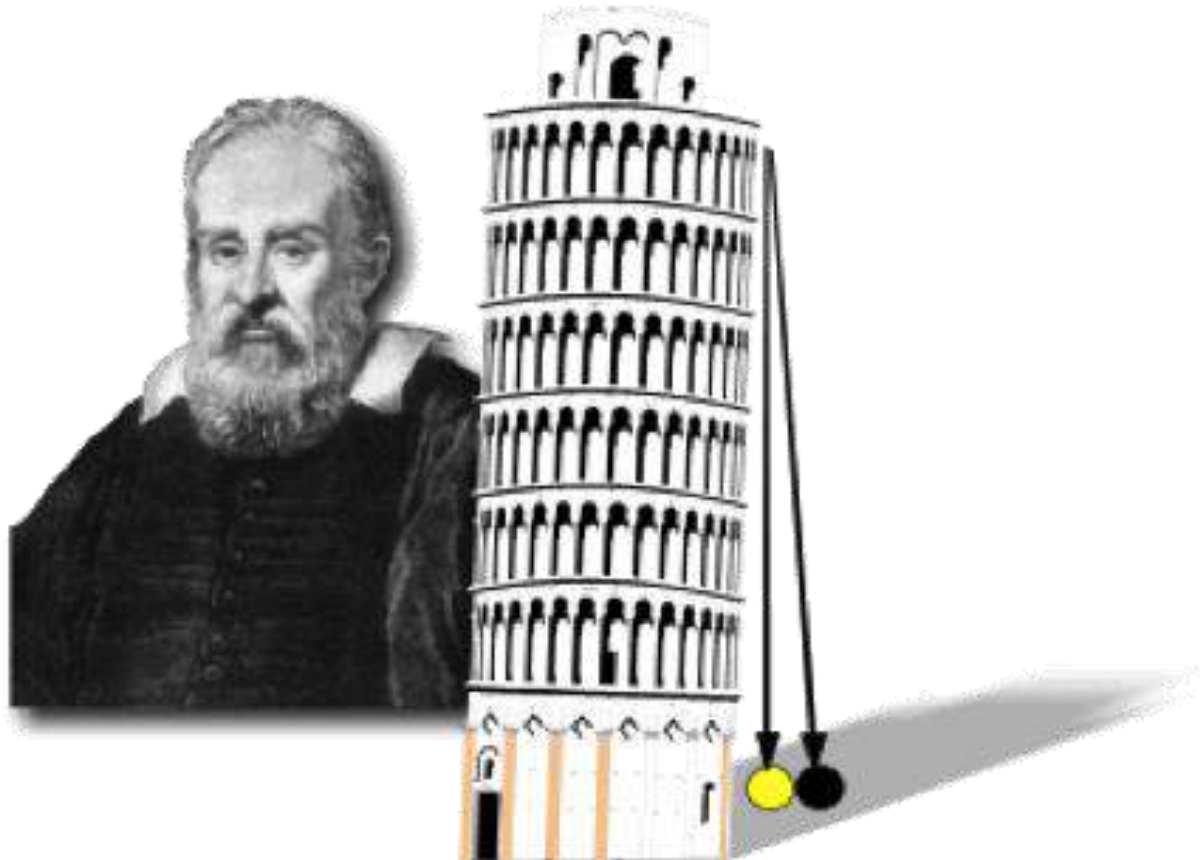
# Brahe, Kepler- 1600

- Heliocentrism, elliptical Orbits



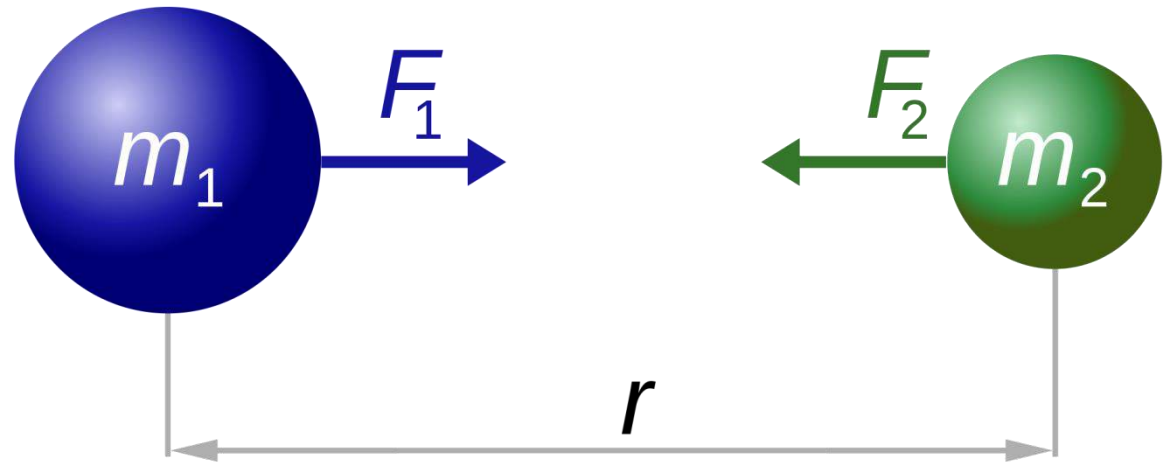
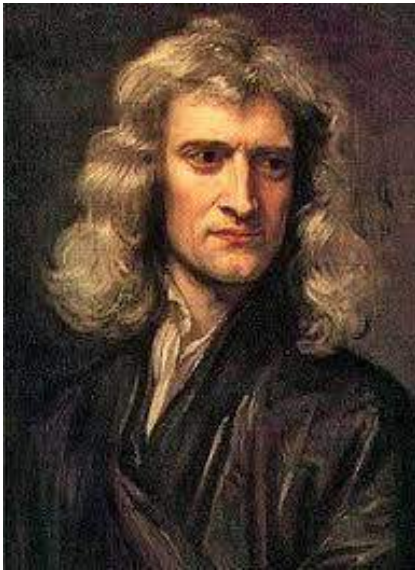
# Galileo - 1600

- Bodies fall with the same speed, **independently** from their **weight**.



# Newton - 1700

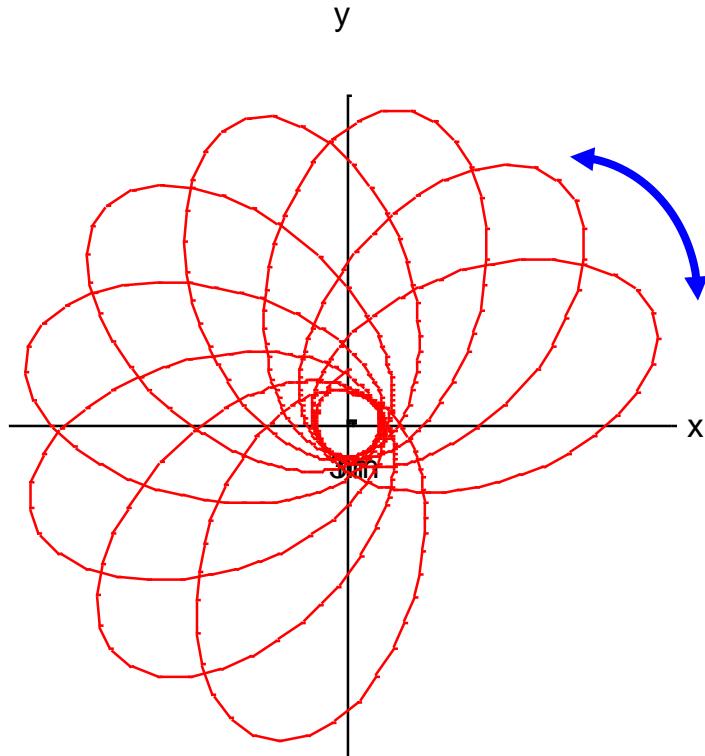
- Law of Universal Gravitation:  
All bodies (either apples or planets) **attract mutually**.  
First time that **gravity is related to astronomy**



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

# Mercury perihelion - 1859

- The true orbits of planets, even if seen from the SUN are not ellipses. They are rather curves of this type:*



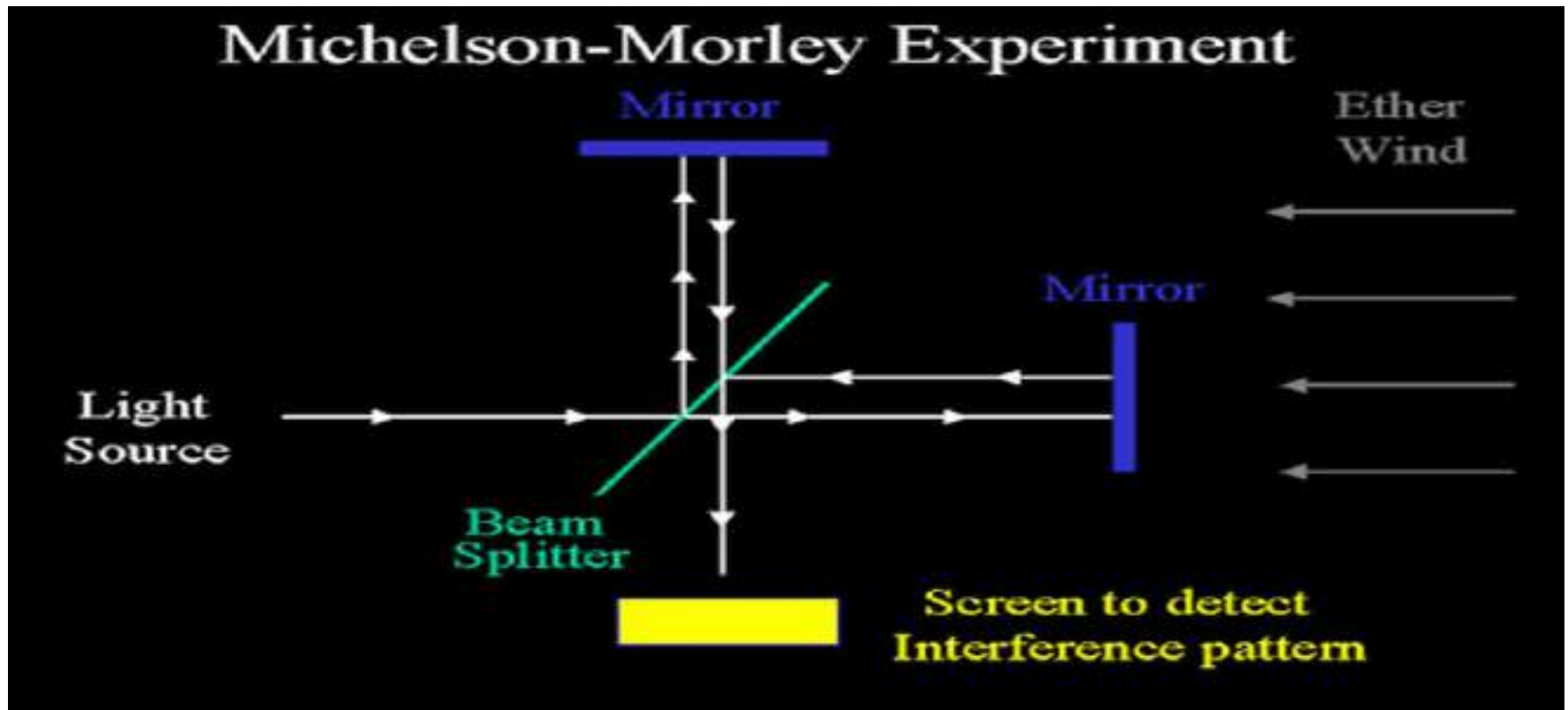
This angle is the perihelion advance, predicted by G.R.

For the planet Mercury it is

$\Delta\varphi = 43''$  of arc per century

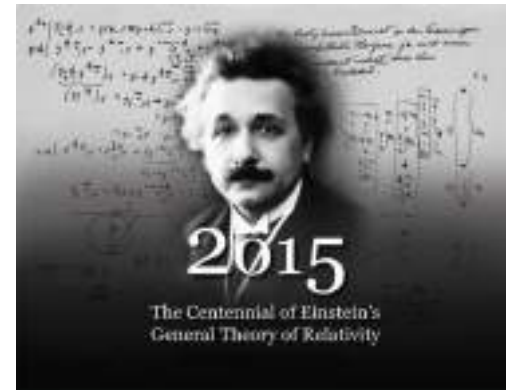


# Michelson–Morley experiment - 1887



# General Relativity

- Einstein 1915: **General Relativity**:



energy-momentum source of spacetime Curvature

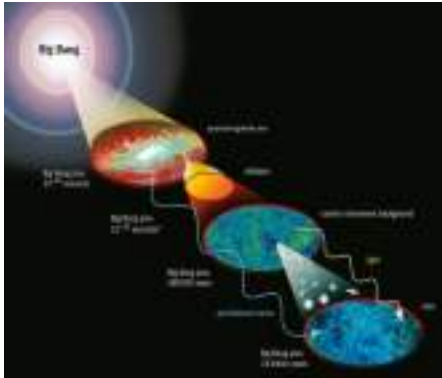
$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] + \int d^4x L_m(g_{\mu\nu}, \psi)$$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

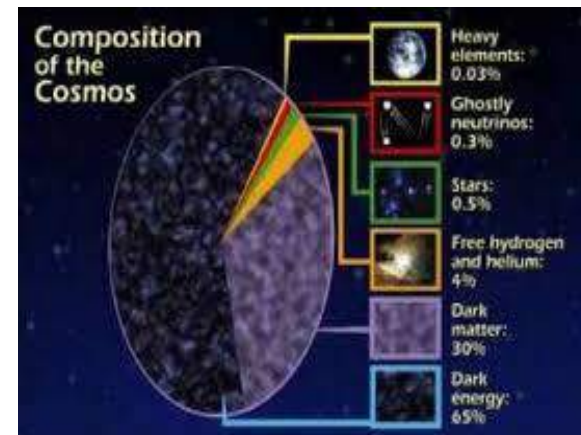
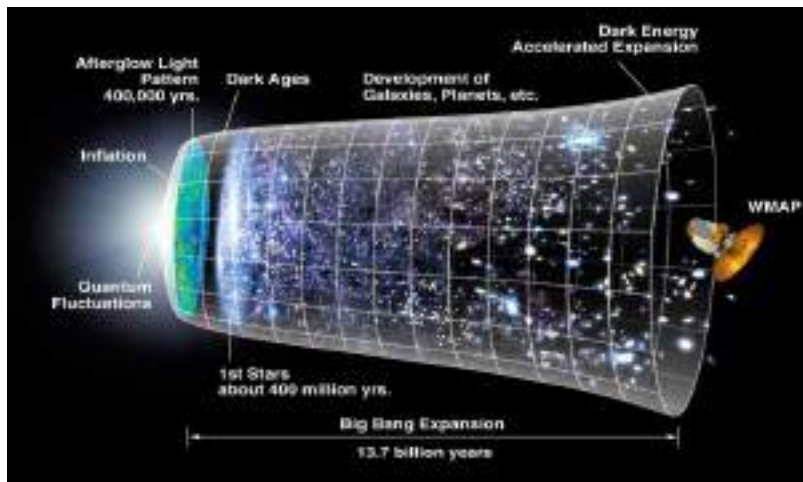
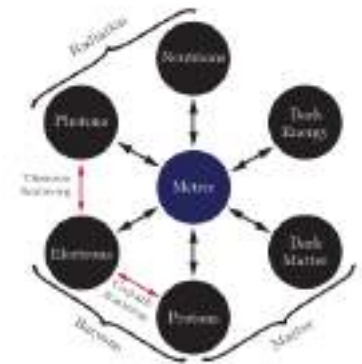
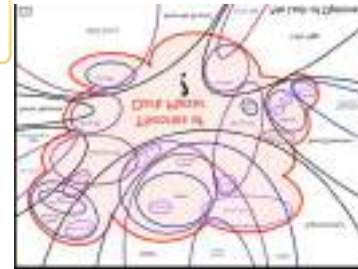
$$\text{with } T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta L_m}{\delta g_{\mu\nu}}$$

# Summary of 20<sup>th</sup> century Observations

## The Universe history:



mass = +2.3 GeV/c <sup>2</sup>	+1.275 GeV/c <sup>2</sup>	+173.17 GeV/c <sup>2</sup>	0	+126 GeV/c <sup>2</sup>
charge = 2/3	2/3	2/3	0	0
spin = 1/2	1/2	1/2	1	0
<b>u</b>	<b>c</b>	<b>t</b>	<b>g</b>	<b>H</b>
up	charm	top	gluon	Higgs boson
<b>QUARKS</b>				
+4.8 MeV/c <sup>2</sup>	+95 MeV/c <sup>2</sup>	+4.18 GeV/c <sup>2</sup>	0	0
-1/3	-1/3	-1/3	0	0
1/2	1/2	1/2	1	1
<b>d</b>	<b>s</b>	<b>b</b>	<b>γ</b>	<b>Z</b>
down	strange	bottom	photon	Z boson
<b>LEPTONS</b>				
0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	0	0
-1	-1	-1	0	0
1/2	1/2	1/2	1	1
<b>e</b>	<b>μ</b>	<b>τ</b>	<b>Z</b>	<b>W</b>
electron	muon	tau	Z boson	W boson
<b>GAUGE BOSONS</b>				
+2.2 eV/c <sup>2</sup>	+0.17 MeV/c <sup>2</sup>	+15.5 MeV/c <sup>2</sup>	80.4 GeV/c <sup>2</sup>	0
0	0	0	0	0
1/2	1/2	1/2	1	1
<b>ν<sub>e</sub></b>	<b>ν<sub>μ</sub></b>	<b>ν<sub>τ</sub></b>	<b>W</b>	<b>W</b>
electron neutrino	muon neutrino	tau neutrino	W boson	W boson



# Standard Model of Cosmology

## $\Lambda$ CDM Paradigm + Inflation

$$H(t)^2 + \frac{k}{a(t)^2} = \frac{8\pi G}{3} [\rho_{dm}(t) + \rho_b(t) + \rho_r(t)] + \frac{\Lambda}{3}$$

$$w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1$$

$$\dot{H}(t) - \frac{k}{a(t)^2} = -4\pi G [\rho_{dm}(t) + p_{dm}(t) + \rho_b(t) + p_b(t) + \rho_r(t) + p_r(t)]$$

$\Lambda$ CDM concordance model is **almost perfect!**

- Describes the **thermal history of the Universe** at the background level
- Epochs of **inflation, radiation, matter, late-time acceleration**

# Cosmology-background

- Homogeneity and isotropy:  $ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$
- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{DE})$$

$$\dot{H} = -4\pi G (\rho_m + p_m + \rho_{DE} + p_{DE}),$$

(the effective DE sector can be either  $\Lambda$  or any possible modification)

- One must obtain a  $H(z)$  and  $\Omega_m(z)$  and  $w_{DE}(z)$  in agreement with observations (SNIa, BAO, CMB shift parameter,  $H(z)$  etc)

# Cosmology-perturbations

- **Perturbation evolution:**  $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta \approx 0$  where  $\delta \equiv \delta\rho/\rho$   
 where  $G_{\text{eff}}(z, k)$  is the **effective Newton's constant**, given by

$$\nabla^2 \phi \approx 4\pi G_{\text{eff}} \rho \delta.$$

under the scalar **metric perturbation**  $ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)d\vec{x}^2$

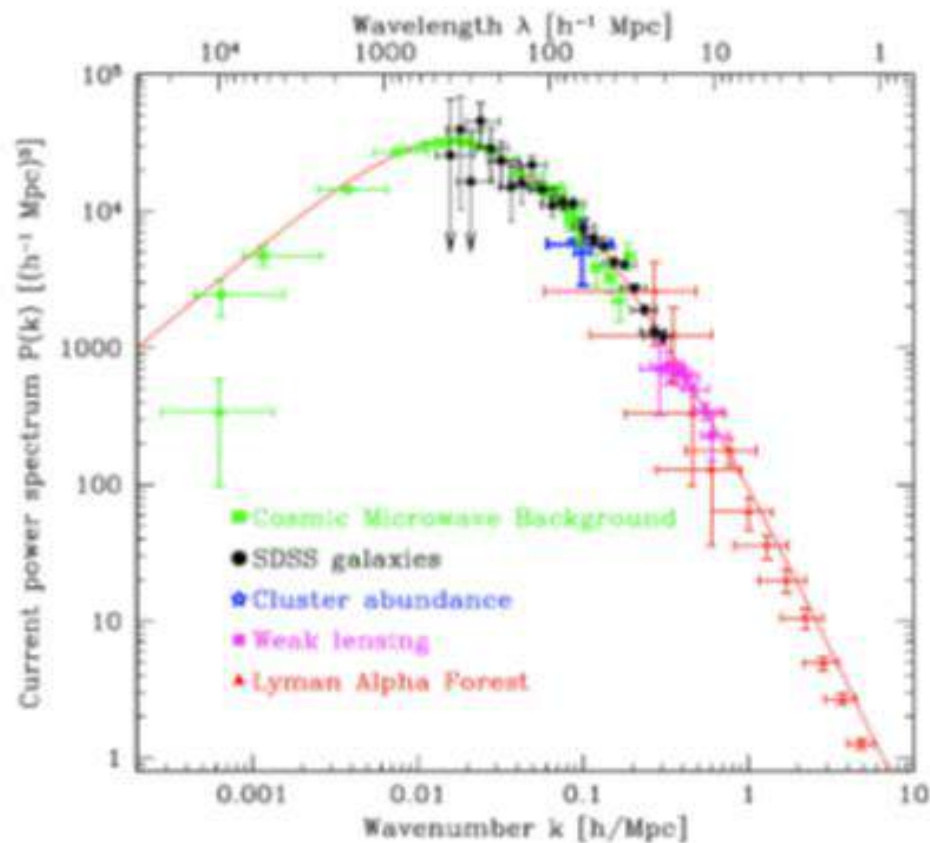
- Hence:  $\delta'' + \left( \frac{(H^2)'}{2H^2} - \frac{1}{1+z} \right) \delta' \approx \frac{3}{2}(1+z) \frac{H_0^2}{H^2} \frac{G_{\text{eff}}(z, k)}{G_N} \Omega_{0m} \delta$

with  $f(a) = \frac{d \ln \delta}{d \ln a}$  the **growth rate**, with  $f(a) = \Omega_m(a)^{\gamma(a)}$  and  $\Omega_m(a) \equiv \frac{\Omega_{0m} a^{-3}}{H(a)^2/H_0^2}$

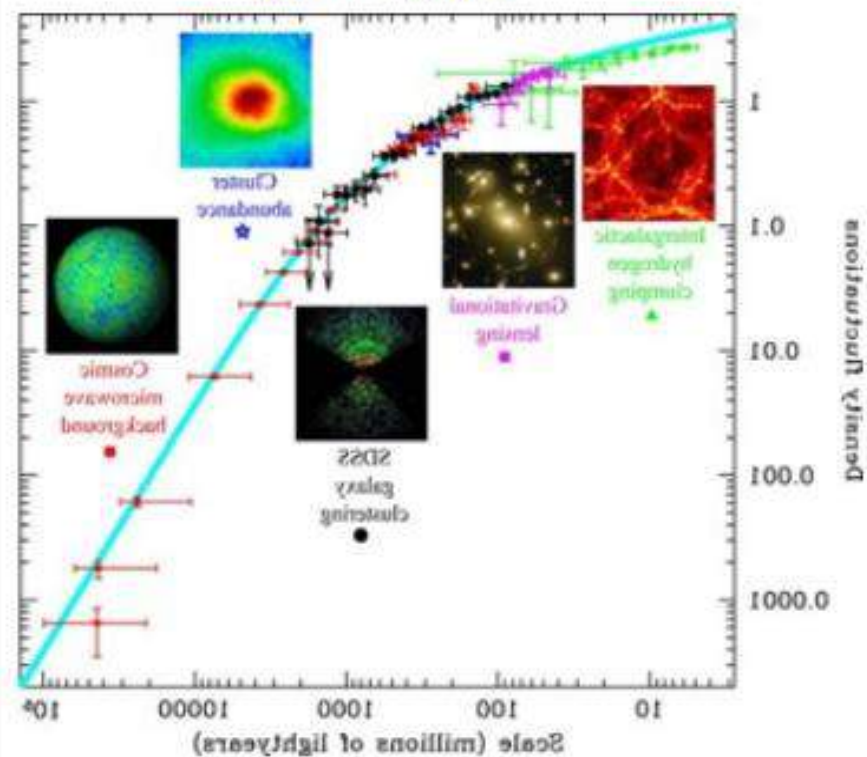
- One can define the **observable**:  $f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_8}{\delta(1)} a \delta'(a)$

with  $\sigma(a) = \sigma_8 \frac{\delta(a)}{\delta(1)}$  the z-dependent rms fluctuations of the linear density field within spheres of radius  $R = 8h^{-1}\text{Mpc}$ , and  $\sigma_8$  its value today.

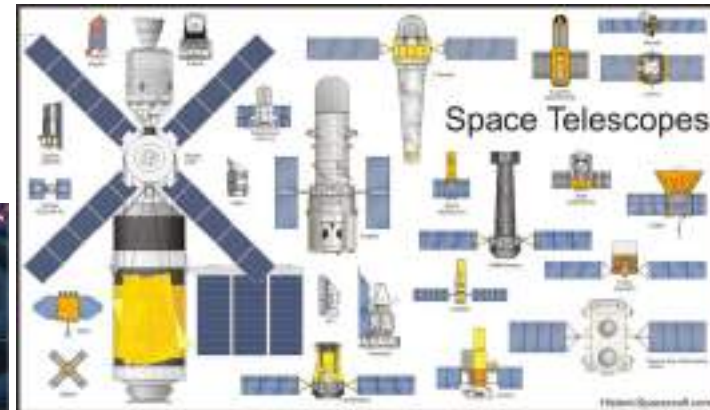
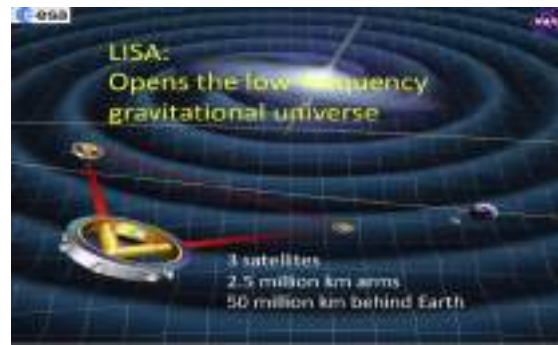
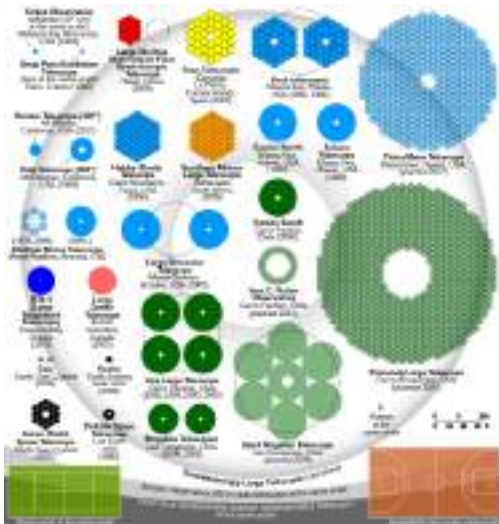
# Matter Density Fluctuation Power Spectrum



A different convention:  
plot  $P(k)k^3$



# Cosmology in the 21<sup>st</sup> century





# Issues of $\Lambda$ CDM Paradigm

- 1) General Relativity is non-renormalizable. It cannot get quantized.
- 2) The cosmological-constant problem.
- 3) How to describe primordial universe (inflation)
- 4) Physics of Dark Matter
- 5) A huge amount of accumulating data suggest possible tensions:

$H_0$ ,  $f\sigma_8$

Challenges for  $\Lambda$ CDM Beyond  $H_0$  and  $S_8$

- A. The  $A_{kin}$  Anomaly in the CMB Angular Power Spectrum
- B. Hints for a Closed Universe from Planck Data
- C. Large-Angular-Scale Anomalies in the CMB Temperature and Polarization
  - 1. The Lack of Large-Angle CMB Temperature Correlations
  - 2. Hemispherical Power Asymmetry
  - 3. Quadrupole and Octopole Anomalies
  - 4. Point-Parity Anomaly
  - 5. Variation in Cosmological Parameters Over the Sky
  - 6. The Cold Spot
  - 7. Explaining the Large-Angle Anomalies
  - 8. Predictions and Future Testability
  - 9. Summary
- D. Abnormal Oscillations of Best Fit Parameter Values
- E. Anomalously Strong ISW Effect
- F. Cosmic Dipoles
  - 1. The  $\alpha$  Dipole
  - 2. Galaxy Cluster Anisotropies and Anomalous Bulk Flows
  - 3. Radio Galaxy Cosmic Dipole
  - 4. QSO Cosmic Dipole and Polarisation Alignments
  - 5. Dipole in SNIa
  - 6. Emergent Dipole in  $H_0$
  - 7. CMB Dipole: Intrinsic Versus Kinematic?
- G. The Ly- $\alpha$  Forest BAO and CMB Anomalies
  - 1. The Ly- $\alpha$  Forest BAO Anomaly
  - 2. Ly- $\alpha$  - Planck 2018 Tension in  $n_s - \Omega_m$
- H. Parity Violating Rotation of CMB Linear Polarization
  - 1. The Lithium Problem
- J. Quasars Hubble Diagram Tension with Planck- $\Lambda$ CDM
- K. Oscillating Force Signals in Short Range Gravity Experiments
- L.  $\Lambda$ CDM and the Dark Matter Phenomenon at Galactic Scales

[L. Perivolaropoulos , F. Scara, New Astron.Rev (2022), 2105.05208 [astro-ph.CO]]

# H0 tension

- **Tension ( $5\sigma$ !)** between the **data** (direct measurements) and **Planck/ $\Lambda$ CDM** (indirect measurements). The data indicate a **lack of "gravitational power"**.

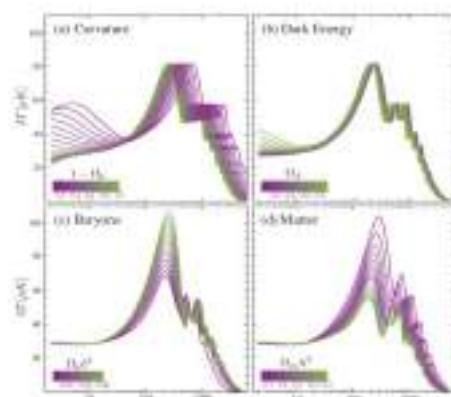
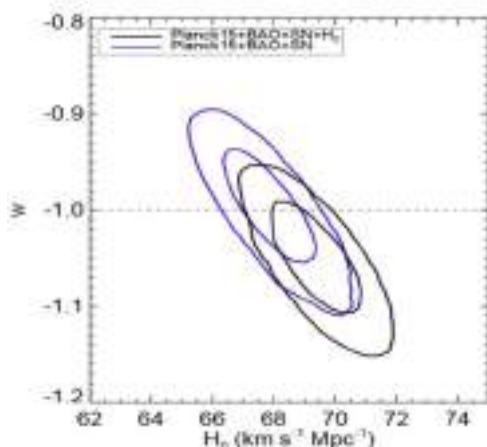
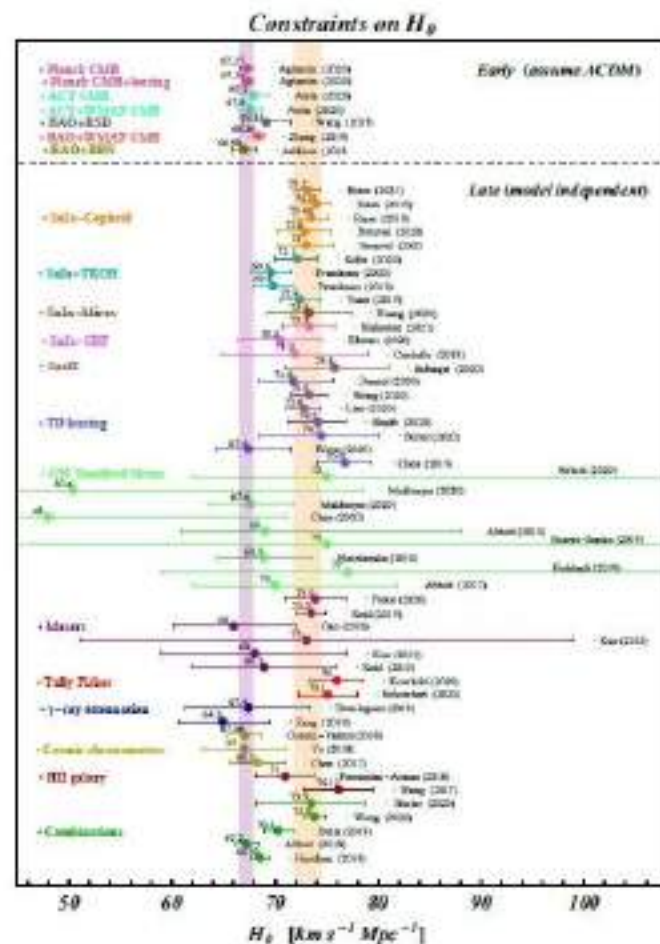


Figure 26: The CMB power spectra as a function of cosmological parameters.

[Riess et al, Astrophys.J 826]



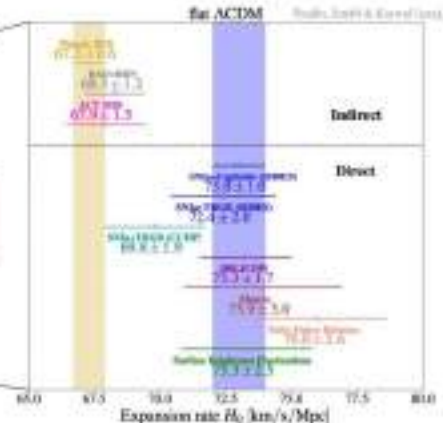
[Abdalla et al, JHEAp (2022)] 18

## Current status

$H_0$  measured / inferred using many techniques

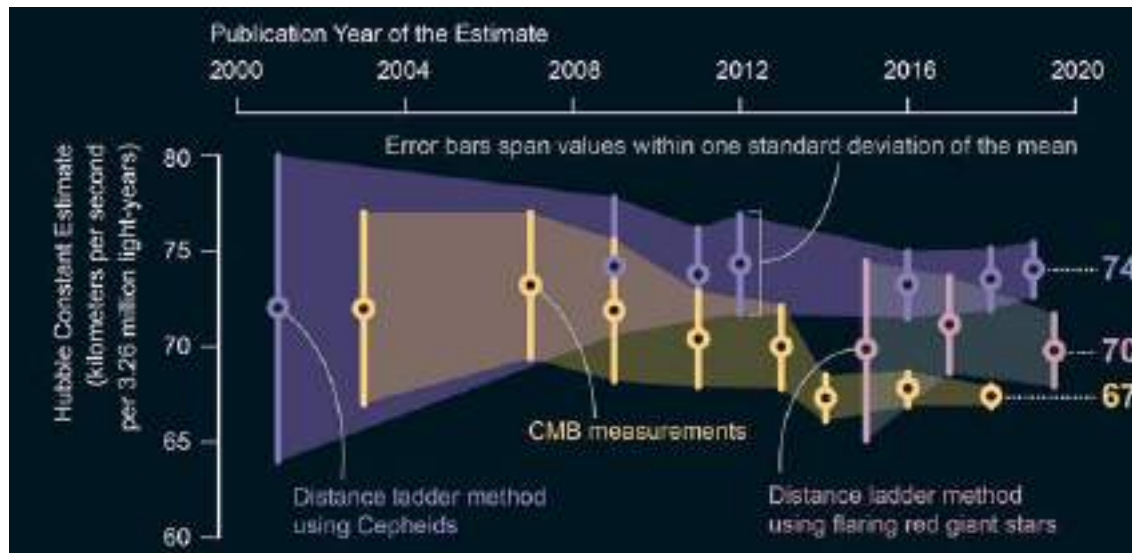


Current tension somewhere between  $\sim 5 - 7\sigma$



# H0 tension

- **Tension** between the **data** (direct measurements) and **Planck/ $\Lambda$ CDM** (indirect measurements). This tension could be due to **systematics**.
- If not systematics then we may need **changes in  $\Lambda$ CDM** in **early** or **late** time behavior.  **$5\sigma$**  seems to be very serious!



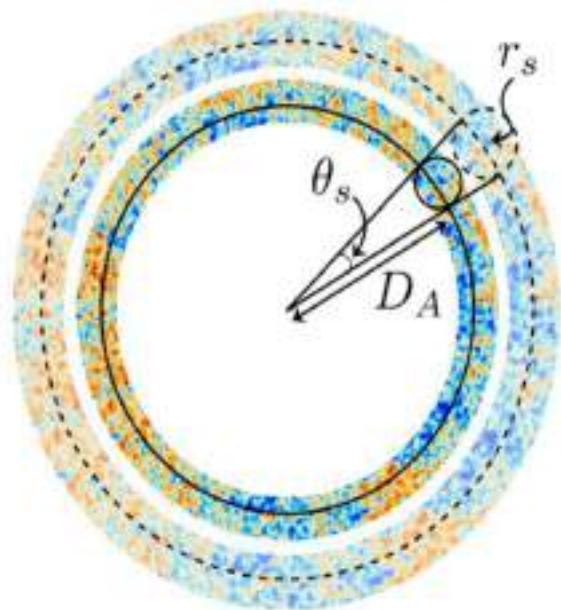
- Change early or late Universe physics. **Higher number of effective relativistic species, dynamical dark energy, non-zero curvature**, etc.
- The data indicate **a lack of "gravitational power"**. **Modified Gravity**.

# Restoring cosmological concordance

*Is LCDM Wrong?*

$$\theta_s = \frac{r_s}{D_A}$$

*0.04% precision*



$$r_s \propto \int_0^{t_{\text{recom}}} dt \frac{c_s(t)}{\rho(t)}$$

$$D_A \propto \frac{1}{H_0} \int_{t_{\text{recom}}}^{t_{\text{today}}} dt \frac{1}{\rho(t)}$$

## How do we increase H<sub>0</sub>?

Decrease sound horizon ( $r_s$ )

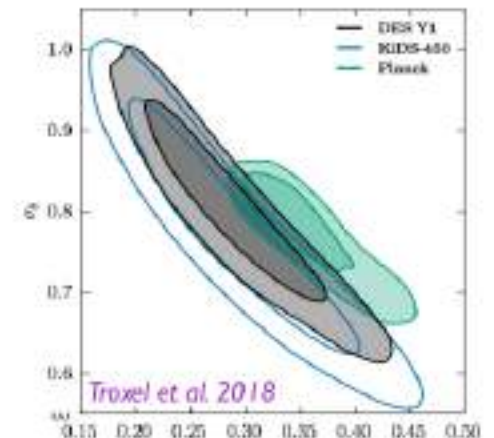
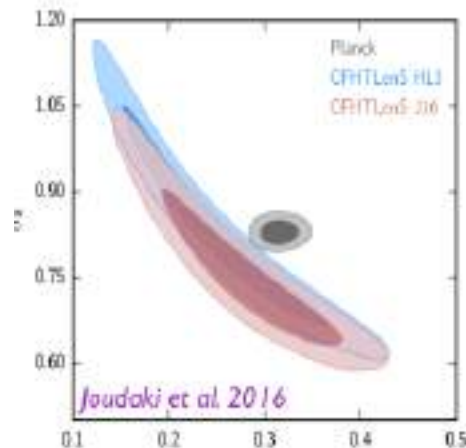
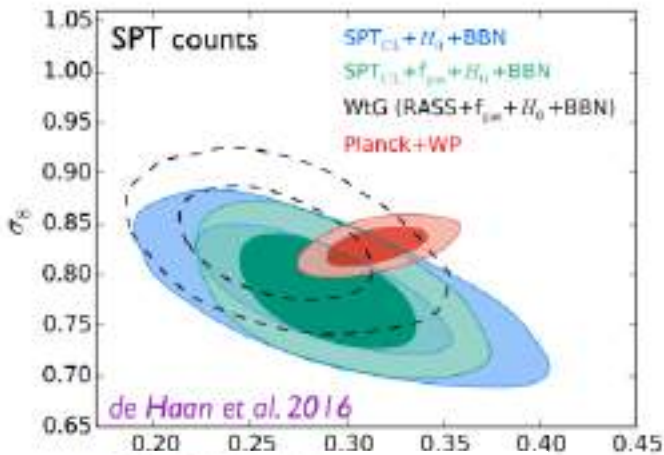
Increase integral in angular diameter distance ( $D_A$ )

*“Early time solutions”*

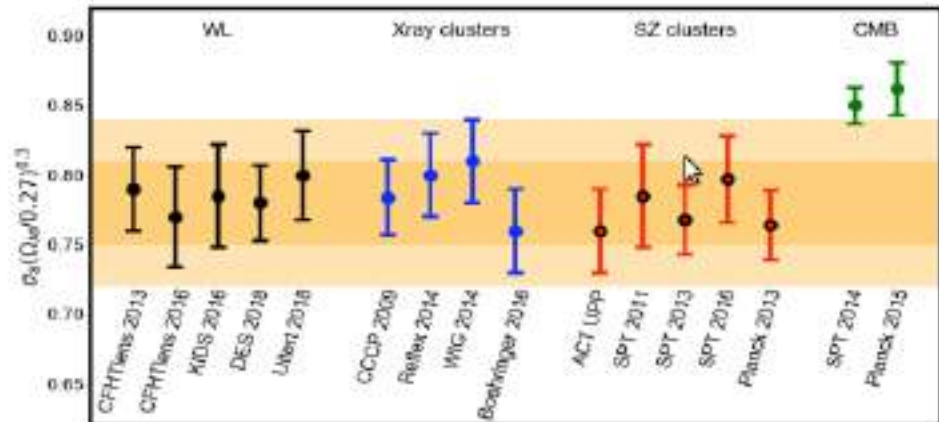
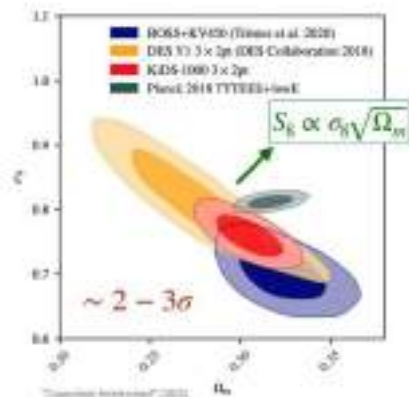
*“Late time solutions”*

# S8 Tension

- **Tension** between **direct data** and **Planck/ $\Lambda$ CDM estimation**. The data indicate **less matter clustering** in structures at intermediate-small cosmological scales.



The S8 Tension



# S8 Tension

TABLE II: A compilation of RSD data that we found published from 2006 since 2018.

Index	Dataset	$\alpha$	$\beta(\%)$	Refs	Year	Statistical Cosmology
1	SDSS-LRG	0.35	$0.460 \pm 0.060$	[75]	30 October 2009	$(\Omega_{\text{m}}, H_0, \sigma_8) = (0.27, 0, 0.750)[10]$
2	VV104	0.77	$0.460 \pm 0.18$	[75]	6 October 2010	$(\Omega_{\text{m}}, H_0, \sigma_8) = (0.25, 0, 0.730)$
3	SDSS-DR5	0.17	$0.510 \pm 0.060$	[77, 78]	6 October 2010	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.2, 0, 0.73)$
4	2MAS	0.02	$0.214 \pm 0.018$	[79, 78]	12 November 2010	$(H_{\text{m}}, \Omega_b, \sigma_8) = (0.240, 0, 0.46)$
5	IRIS+DRAS	0.02	$0.298 \pm 0.067$	[79, 78]	20 October 2011	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.3, 0, 0.814)$
6	SDSS-LRG-200	0.35	$0.3512 \pm 0.0683$	[50]	3 December 2011	$(\Omega_{\text{m}}, H_0, \sigma_8) = (0.270, 0, 0.8)$
7	SDSS-LRG-200	0.37	$0.4082 \pm 0.0279$	[60]	3 December 2011	
8	SDSS-LRG-00	0.25	$0.3000 \pm 0.0600$	[60]	3 December 2011	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.270, 0, 0.8)$
9	SDSS-LRG-00	0.37	$0.4020 \pm 0.0604$	[60]	3 December 2011	
10	WiggleZ	0.44	$0.413 \pm 0.080$	[66]	12 June 2012	$(H_{\text{m}}, \theta, \sigma_8) = (0.27, 0.71, 0.8)$
11	WiggleZ	0.60	$0.290 \pm 0.093$	[66]	12 June 2012	$\Omega_b = \Omega_b(3.3)$
12	WiggleZ	0.72	$0.427 \pm 0.072$	[68]	12 June 2012	
13	SDSS-DC08	0.007	$0.427 \pm 0.065$	[61]	4 July 2012	$(H_{\text{m}}, \Omega_b, \sigma_8) = (0.27, 0, 0.75)$
14	SDSS-DC08	0.30	$0.497 \pm 0.028$	[61]	11 August 2012	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.25, 0, 0.804)$
15	SDSS-DC08	0.40	$0.415 \pm 0.041$	[62]	11 August 2012	
16	SDSS-DC08	0.60	$0.427 \pm 0.043$	[62]	11 August 2012	
17	SDSS-DC08	0.60	$0.423 \pm 0.087$	[63]	11 August 2012	
18	Vipers	0.80	$0.470 \pm 0.090$	[64]	5 July 2013	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.25, 0, 0.82)$
19	SDSS-DR7-LRG	0.35	$0.428 \pm 0.060$	[64]	8 August 2013	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.25, 0, 0.800)[10]$
20	GAMA	0.18	$0.300 \pm 0.060$	[65]	22 September 2013	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.27, 0, 0.8)$
21	GAMA	0.35	$0.440 \pm 0.060$	[65]	22 September 2013	
22	RCOS-LGWZ	0.32	$0.384 \pm 0.095$	[67]	17 December 2013	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.274, 0, 0.8)$
23	SDSS DR10 and DR11	0.32	$0.45 \pm 0.10$	[67]	17 December 2013	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.274, 0, 0.8)[10]$
24	SDSS DR10 and DR11	0.37	$0.417 \pm 0.0425$	[67]	17 December 2013	
25	SDSS-MG2	0.15	$0.490 \pm 0.147$	[68]	30 January 2015	$(H_{\text{m}}, \theta, \sigma_8) = (0.31, 0.67, 0.83)$
26	SDSS-DR7c	0.15	$0.395 \pm 0.130$	[69]	18 June 2015	$(H_{\text{m}}, \Omega_b, \sigma_8) = (0.2, 0, 0.85)[10]$
27	FastSound	1.40	$0.483 \pm 0.118$	[69]	20 November 2015	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.27, 0, 0.82)[10]$
28	RCOS-CMASS	0.59	$0.488 \pm 0.090$	[70]	8 July 2016	$(H_{\text{m}}, \theta, \sigma_8) = (0.307115, 0.6777, 0.8258)$
29	RCOS DR12	0.38	$0.497 \pm 0.0425$	[71]	11 July 2016	$(H_{\text{m}}, \theta, \sigma_8) = (0.31, 0, 0.8)$
30	RCOS DR12	0.51	$0.428 \pm 0.038$	[72]	11 July 2016	
31	RCOS DR12	0.81	$0.498 \pm 0.094$	[72]	11 July 2016	
32	RCOS DR12	0.38	$0.477 \pm 0.051$	[66]	11 July 2016	$(H_{\text{m}}, \theta, \sigma_8) = (0.31, 0.670, 0.8)$
33	RCOS DR12	0.51	$0.483 \pm 0.050$	[66]	11 July 2016	
34	RCOS DR12	0.61	$0.410 \pm 0.043$	[66]	11 July 2016	
35	Vipers vt	0.70	$0.440 \pm 0.040$	[66]	20 October 2016	$(H_{\text{m}}, \sigma_8) = (0.208, 0.8148)$
36	Vipers vt	1.65	$0.280 \pm 0.060$	[66]	26 October 2016	
37	RCOS-LGWZ	0.32	$0.497 \pm 0.095$	[66]	26 October 2016	$(H_{\text{m}}, \Omega_b, \sigma_8) = (0.31, 0, 0.8475)$
38	RCOS-CMASS	0.57	$0.428 \pm 0.028$	[66]	26 October 2016	
39	Vipers	0.727	$0.290 \pm 0.0260$	[67]	21 November 2016	$(H_{\text{m}}, H_0, \sigma_8) = (0.31, 0, 0.7)$
40	SDSS+Data	0.02	$0.428 \pm 0.040$	[68]	20 November 2016	$(H_{\text{m}}, \theta, \sigma_8) = (0.2, 0.802, 0.8)$
41	Vipers	0.6	$0.48 \pm 0.12$	[68]	10 December 2016	$(\Omega_{\text{m}}, \Omega_b, \sigma_8, \sigma_8) = (0.2, 0.045, 0.04, 0.801)[10]$
42	Vipers	0.80	$0.48 \pm 0.10$	[68]	10 December 2016	
43	Vipers P00-2	0.60	$0.350 \pm 0.120$	[100]	10 December 2016	$(H_{\text{m}}, \Omega_b, \sigma_8) = (0.3, 0.045, 0.825)$
44	Vipers P00-2	0.30	$0.400 \pm 0.110$	[100]	10 December 2016	
45	RCOS DR13	0.4	$0.48 \pm 0.12$	[101]	20 December 2016	$(H_{\text{m}}, \sigma_8) = (0.33, 0.66)[10]$
46	2MTF	0.001	$0.200 \pm 0.060$	[102]	14 June 2017	$(H_{\text{m}}, \sigma_8) = (0.121, 0.818)$
47	Vipers P00-2	0.80	$0.45 \pm 0.11$	[103]	21 July 2017	$(H_0, \Omega_{\text{m}}, \theta) = (0.045, 0.26, 0.8)$
48	RCOS DR12	0.34	$0.460 \pm 0.068$	[68]	15 September 2017	$(H_{\text{m}}, \theta, \sigma_8) = (0.307, 0.677, 0.8258)$
49	RCOS DR14	0.30	$0.474 \pm 0.097$	[68]	15 September 2017	
50	RCOS DR12	0.40	$0.473 \pm 0.065$	[68]	15 September 2017	
51	RCOS DR12	0.44	$0.481 \pm 0.070$	[68]	15 September 2017	
52	RCOS DR12	0.48	$0.482 \pm 0.067$	[68]	15 September 2017	
53	RCOS DR12	0.52	$0.485 \pm 0.065$	[68]	15 September 2017	
54	RCOS DR12	0.60	$0.493 \pm 0.067$	[68]	15 September 2017	
55	RCOS DR12	0.59	$0.481 \pm 0.060$	[68]	15 September 2017	
56	RCOS DR12	0.64	$0.486 \pm 0.070$	[68]	15 September 2017	
57	SDSS DR7	0.1	$0.276 \pm 0.028$	[104]	12 December 2017	$(\Omega_{\text{m}}, H_0, \sigma_8) = (0.282, 0.040, 0.817)$
58	SDSS-IV	1.02	$0.420 \pm 0.070$	[105]	8 January 2018	$(H_{\text{m}}, \Omega_b, \sigma_8) = (0.2649, 0.02285, 0.7)$
59	SDSS-IV	1.50	$0.304 \pm 0.070$	[106]	8 January 2018	$(\Omega_{\text{m}}, \Omega_b, \sigma_8) = (0.21, 0.038, 0.8091)$
60	SDSS-IV	0.678	$0.279 \pm 0.170$	[107]	9 January 2018	$(\Omega_{\text{m}}, \Omega_b) = (0.21, 0.8)$
61	SDSS-IV	1.23	$0.285 \pm 0.060$	[107]	9 January 2018	
62	SDSS-IV	1.526	$0.242 \pm 0.070$	[107]	9 January 2018	
63	SDSS-IV	1.084	$0.364 \pm 0.100$	[107]	9 January 2018	

- Model Dependence: Distance to galaxies is not measured directly, so a cosmological model is assumed in order to infer distances ( $\Lambda$ CDM with different parameters).
- Double counting: Some data points correspond to the same sample of galaxies analyzed by different groups/methods etc.

[Kazantzidis, Perivolaropoulos, PRD97]

# Tension2 – $f\sigma_8$

- Tension between the data and Planck/ $\Lambda$ CDM.
- This tension could be due to systematics.
- If not systematics, the data less matter clustering in structures at intermediate-small cosmological scales (expressed as smaller  $\Omega_m$  at  $z < 0.6$ , or smaller  $\sigma_8$ , or  $w_{DE} < -1$ ).
- It could be reconciled by a mechanism that reduces the rate of clustering between recombination and today: Hot Dark Matter, Dark Matter that clusters differently at small scales, or Modified Gravity.

# Possible Solutions of H0 and S8 tensions

tension $\leq 1\sigma$ "Excellent models"	tension $\leq 2\sigma$ "Good models"	tension $\leq 3\sigma$ "Promising models"
Dark energy in extended parameter spaces [289] Dynamical Dark Energy [309] Metastable Dark Energy [314] PEDE [392, 394] Elaborated Vacuum Metamorphosis [400–402] IDE [314, 636, 637, 639, 652, 657, 661–663] Self-interacting sterile neutrinos [711] Generalized Chaplygin gas model [744] Galileon gravity [876, 882] Power Law Inflation [966] $f(\mathcal{T})$ [818]	Early Dark Energy [235] Phantom Dark Energy [11] Dynamical Dark Energy [11, 281, 309] GEDE [397] Vacuum Metamorphosis [402] IDE [314, 653, 656, 661, 663, 670] Critically Emergent Dark Energy [997] $f(\mathcal{T})$ gravity [814] Über-gravity [59] Reconstructed PPS [978]	Early Dark Energy [229] Decaying Warm DM [474] Neutrino-DM Interaction [506] Interacting dark radiation [517] Self-Interacting Neutrinos [700, 701] IDE [656] Unified Cosmologies [747] Scalar-tensor gravity [856] Modified recombination [986] Super $\Lambda$ CDM [1007] Coupled Dark Energy [650]
Early Dark Energy [228, 235, 240, 250] Exponential Acoustic Dark Energy [259] Phantom Crossing [315] Late Dark Energy Transition [317] Metastable Dark Energy [314] PEDE [394] Vacuum Metamorphosis [402] Elaborated Vacuum Metamorphosis [401, 402] Sterile Neutrinos [433] Decaying Dark Matter [481] Neutrino-Majoron Interactions [509] IDE [637, 639, 657, 661] DM - Photon Coupling [685] $f(\mathcal{T})$ gravity theory [812] BD- $\Lambda$ CDM [851] Über-Gravity [59] Galileon Gravity [875] Unimodular Gravity [890] Time Varying Electron Mass [990] $\Lambda$ CDM [995] Ginzburg-Landau theory [996] Lorentzian Quintessential Inflation [979] Holographic Dark Energy [351]	Early Dark Energy [212, 229, 236, 263] Rock 'n' Roll [242] New Early Dark Energy [247] Acoustic Dark Energy [257] Dynamical Dark Energy [309] Running vacuum model [332] Bulk viscous models [340, 341] Holographic Dark Energy [350] Phantom Braneworld DE [378] PEDE [391, 392] Elaborated Vacuum Metamorphosis [401] IDE [659, 670] Interacting Dark Radiation [517] Decaying Dark Matter [471, 474] DM - Photon Coupling [686] Self-interacting sterile neutrinos [711] $f(\mathcal{T})$ gravity theory [817] Über-Gravity [871] VCDM [893] Primordial magnetic fields [992] Early modified gravity [859] Bianchi type I spacetime [999] $f(\mathcal{T})$ [818]	DE in extended parameter spaces [289] Dynamical Dark Energy [281, 309] Holographic Dark Energy [350] Swampland Conjectures [370] MEDE [399] Coupled DM - Dark radiation [534] Decaying Ultralight Scalar [538] BD- $\Lambda$ CDM [852] Metastable Dark Energy [314] Self-Interacting Neutrinos [700] Dark Neutrino Interactions [716] IDE [634–636, 653, 656, 663, 669] Scalar-tensor gravity [855, 856] Galileon gravity [877, 881] Nonlocal gravity [886] Modified recombination [986] Effective Electron Rest Mass [989] Super $\Lambda$ CDM [1007] Axi-Higgs [991] Self-Interacting Dark Matter [479] Primordial Black Holes [545]



# Possible Solutions of $H_0$ and S8 tensions

## Early-Time Alternative Proposed Models

1. Axion Monodromy
2. Early Dark Energy
3. Extra Relativistic Degrees of Freedom
4. Modified Recombination History
5. New Early Dark Energy

## Late-Time Alternative Proposed Models

1. Bulk Viscous Models
2. Chameleon Dark Energy
3. Clustering Dark Energy
4. Diffusion Models
5. Dynamical Dark Energy
6. Emergent Dark Energy
7. Graduated Dark Energy - AdS to dS Transition in the Late Universe
8. Holographic Dark Energy
9. Interacting Dark Energy
10. Quintessence Models and their Various Extensions
11. Running Vacuum Models
12. Time-Varying Gravitational Constant
13. Vacuum Metamorphosis

## Modified Gravity Models

1. Effective Field Theory Approach to Dark Energy and Modified Gravity
2.  $f(T)$  Gravity
3. Horndeski Theory
4. Quantum Conformal Anomaly Effective Theory and Dynamical Vacuum Energy
5. Ultra-Late Time Gravitational Transitions

## Beyond the FLRW Framework

1. Cosmological Fitting and Averaging Problems
2. Data Analysis in an Universe with Structure: Accounting for Regional Inhomogeneity and Anisotropy
3. Local Void Scenario

## Specific Solutions Assuming FLRW

1. Active and Sterile Neutrinos
2. Cannibal Dark Matter
3. Decaying Dark Matter
4. Dynamical Dark Matter
5. Extended Parameter Spaces Involving  $A_{\text{kin}}$
6. Cosmological Scenario with Features in the Primordial Power Spectrum
7. Interacting Dark Matter
8. Quantum Landscape Multiverse
9. Quantum Fisher Cosmology
10. Quartessence
11. Scaling Symmetry and a Mirror Sector
12. Self-Interacting Neutrinos
13. Self-Interacting Sterile Neutrinos
14. Soft Cosmology
15. Two-Body Decaying Cold Dark Matter into Dark Radiation and Warm Dark Matter

**Cosmology Intertwined:  
A Review of the Particle Physics, Astrophysics, and Cosmology  
Associated with the Cosmological Tensions and Anomalies**

Elcio Abdalla,<sup>1</sup> Guillermo Franco Abellin,<sup>2</sup> Amin Abenbrahim,<sup>3</sup> Adriano Agnello,<sup>4</sup> Özgür Aksoy,<sup>5</sup> Vishar Akrami,<sup>6,7,8,9</sup> George Aksoy,<sup>10</sup> Daniel Aloni,<sup>11</sup> Luca Amendola,<sup>12</sup> Luis A. Anchordoqui,<sup>13,14,15</sup> Richard I. Anderson,<sup>16</sup> Niko Ando,<sup>17</sup> Martin Asgari,<sup>18,19</sup> Maria Ballardini,<sup>20,21,22,23</sup> Vernon Baerje,<sup>24</sup> Spyros Basilakos,<sup>25,26</sup> Romulo C. Batista,<sup>27</sup> Elia S. Bertoldi,<sup>28,29</sup> Richard Brito,<sup>30</sup> Mikael Bonetti,<sup>31,32</sup> David Bousoff,<sup>33,34,35</sup> Asher Berlin,<sup>36</sup> Paulo de Bernardis,<sup>37,38</sup> Emanuele Bertl,<sup>39</sup> Dobrosława Bilenska,<sup>38,40</sup> Simon Birnst,<sup>41</sup> John P. Blakeslee,<sup>42</sup> Kimberly K. Boddy,<sup>43</sup> Clodio R. Bom,<sup>44,45</sup> Alexander Bonilla,<sup>46</sup> Nicola Borghi,<sup>46,47</sup> François Bouchet,<sup>48</sup> Matteo Draglino,<sup>49,50</sup> Thomas Buchert,<sup>51</sup> Elizabeth Buckley-Goer,<sup>51,52</sup> Erminda Calabrese,<sup>53</sup> Robert R. Caldwell,<sup>54</sup> David Casanova,<sup>55</sup> Salvatore Capozziello,<sup>56,57</sup> Stefano Casertano,<sup>57</sup> Angela Chen,<sup>58,59</sup> Geoff C.-F. Chen,<sup>60</sup> Hsu-Yu Chen,<sup>61</sup> Jens Chluba,<sup>62</sup> Anton Chulagkin,<sup>63</sup> Michele Cioldi,<sup>60,22</sup> Craig J. Copi,<sup>64</sup> Fred Courbin,<sup>65</sup> Francis-Yan Cyr-Racine,<sup>66</sup> Beona Coma,<sup>64</sup> Maria Dalmati,<sup>65,66,67</sup> Guido D'Amico,<sup>68,69</sup> Anne-Christine Davis,<sup>44,34</sup> Javier de Cruz Pérez,<sup>70</sup> Jaume de Haan,<sup>71</sup> Jacques Delabrouille,<sup>72,73,74,75</sup> Peter B. Denton,<sup>76</sup> Suhail Dhawan,<sup>77</sup> Keith R. Dineen,<sup>78,79</sup> Eleonora Di Valentino,<sup>80,7</sup> Pu Du,<sup>81</sup> Dominique Eckert,<sup>82</sup> Celia Escudella-Ribera,<sup>83</sup> Agnès Farié,<sup>84</sup> Fabio Finelli,<sup>85,86</sup> Pablo Fosalba,<sup>86,87</sup> Wendy L. Freedman,<sup>88</sup> Normi Frusciante,<sup>89</sup> Enrique Gaztañaga,<sup>88,89</sup> William Gao,<sup>88,89</sup> Elena Gómez-Vale,<sup>90</sup> Adria Gómez-Vale,<sup>91</sup> Will Handley,<sup>92,93</sup> Ian Harrison,<sup>94</sup> Lulu He,<sup>95</sup> Dhruv Kumar Haerzi,<sup>96</sup> Alan Heavens,<sup>97</sup> Asta Heinesen,<sup>98</sup> Hendrik Hildebrandt,<sup>99</sup> J. Colin Hill,<sup>97,98</sup> Natalie B. Hogg,<sup>99</sup> Daniel E. Holz,<sup>100,101</sup> Dennis C. Hooper,<sup>102</sup> Nilou Hossainzadeh,<sup>103</sup> Dragana Hutonen,<sup>104,105</sup> Mustapha Ishak,<sup>106</sup> Mikhail M. Ivanov,<sup>107</sup> Andrew H. Jaffe,<sup>7</sup> In Sung Jang,<sup>108</sup> Kristen Janiak,<sup>109</sup> Raul Jimenez,<sup>109,110</sup> Melissa Joseph,<sup>11</sup> Shahab Joutaki,<sup>111,112</sup> Mare Kaminikowicz,<sup>7</sup> Tanvi Karwal,<sup>113</sup> Lavrentios Kazantzidis,<sup>114</sup> Ryan E. Keeley,<sup>115</sup> Michael Klason,<sup>3</sup> Eiichiro Komatsu,<sup>116,118</sup> Léon V.E. Koopmans,<sup>117</sup> Suresh Kumar,<sup>118</sup> Luca Lamagna,<sup>20,49</sup> Ruth Lazkoz,<sup>119</sup> Cheung-Chi Lee,<sup>120</sup> Julien Lesgourgues,<sup>121</sup> Jackson Levi Soid,<sup>122,123</sup> Tiffany R. Lewis,<sup>124</sup> Benjamin L'Huilier,<sup>125</sup> Matteo Luzzo,<sup>126</sup> Roy Maartens,<sup>11,127,128</sup> Lucas M. Macri,<sup>129</sup> Danny Marfatia,<sup>130</sup> Valeria Marra,<sup>131,132,133</sup> Carlos J. A. P. Martins,<sup>130,131</sup> Sívrio Miel,<sup>20,29</sup> Sábina Matorrós,<sup>134,135,136,137</sup> Arindam Majumdar,<sup>138</sup> Alessandro Melchiorri,<sup>28,39</sup> Olga Mena,<sup>140</sup> Laura Merini-Houghton,<sup>141</sup> James Mertens,<sup>142</sup> Dinko Mihaljević,<sup>143,144,145</sup> Yuto Minami,<sup>146</sup> Vitoria Miranda,<sup>148</sup> Cristian Moreno-Pulido,<sup>147</sup> Michele Moroso,<sup>18,37</sup> David F. Mota,<sup>148</sup> Emil Mottola,<sup>51</sup> Simone Moresco,<sup>149</sup> Jessica Muir,<sup>150</sup> Ankan Mukherjee,<sup>151</sup> Sumitpr Mukherjee,<sup>152</sup> Pavel Nasrsky,<sup>153</sup> Pran Nath,<sup>154</sup> Savvas Nesseris,<sup>89</sup> Florian Niedermann,<sup>20</sup> Alessio Notari,<sup>155</sup> Rafael C. Nunes,<sup>156</sup> Eoin Ó Colgáin,<sup>157,158</sup> Kayla A. Owens,<sup>91</sup> Enzo Özeller,<sup>3</sup> Francesco Pace,<sup>159,160</sup> Andrius Palishausas,<sup>161,162</sup> Antonella Palmese,<sup>163</sup> Supriya Pan,<sup>164</sup> Daniela Panketi,<sup>85,22</sup> Santiago E. Perez Berglaffa,<sup>165</sup> Leonidas Perivolaropoulos,<sup>166</sup> Dominic W. Posso,<sup>166,167</sup> Valeria Pettorino,<sup>168</sup> Oliver H. E. Philcox,<sup>169,167</sup> Levan Paguisan,<sup>170</sup> Vivian Paoletti,<sup>2</sup> Gaspari Pardo,<sup>80</sup> Marco Pavri,<sup>171</sup> Mark J. Reid,<sup>172</sup> Fabrizio Renzi,<sup>173</sup> Adam G. Riess,<sup>37</sup> Virvian I. Saldaña,<sup>174</sup> Paolo Salucci,<sup>175,176</sup> Vincenzo Sabano,<sup>178</sup> Emmanuel N. Saldanha,<sup>86,75,177</sup> Bangalore S. Sathyapal,<sup>178,179,84</sup> Martin Schmidt,<sup>11</sup> Nils Schönberg,<sup>180</sup> Dan Scolnic,<sup>181</sup> Anjan A. Sen,<sup>182,183</sup> Neelima Sehgal,<sup>184</sup> Arman Shafieloo,<sup>185</sup> M.M. Sheikh-Jabbari,<sup>186</sup> Joseph Silk,<sup>187</sup> Alexandria Silvestri,<sup>174</sup> Efrim Sironi,<sup>188</sup> Martin S. Smith,<sup>189</sup> Marcelle Soares-Santos,<sup>18</sup> Juan Solá Peraza,<sup>142</sup> Yu-Yang Songsheng,<sup>81</sup> Jerzy F. Soriano,<sup>11,14</sup> Dennis Stairs,<sup>190</sup> Glenn D. Starkman,<sup>6,7</sup> István Szapudi,<sup>191</sup> Elsa M. Tjebbe,<sup>93</sup> Brooks Tiziano,<sup>191</sup> Emmanouil Treu,<sup>80</sup> Emory Trott,<sup>58</sup> Carsten van de Bruck,<sup>50</sup> J. Alberto Vazquez,<sup>192</sup> Lidia Verde,<sup>193,194</sup> Luca Vismalli,<sup>195</sup> Deng Wang,<sup>196</sup> Jian-Min Wang,<sup>81</sup> Shao-Jiang Wang,<sup>197</sup> Richard Watkins,<sup>198</sup> Scott Watson,<sup>199</sup> John K. Webb,<sup>200</sup> Neal Weiner,<sup>200</sup> Amanda Weltman,<sup>191</sup> Samuel J. Witte,<sup>202</sup> Radwan Wojtak,<sup>6</sup> Anil Kumar Yadav,<sup>203</sup> Weiqiang Yang,<sup>204</sup> Gong-Bo Zhao,<sup>205,206</sup> and Miguel Zumalacabarri.<sup>207</sup>

<sup>1</sup>Instituto de Física, Universidade de São Paulo - C.P. 66218, CEP: 05515-970, São Paulo, Brazil  
<sup>2</sup>Laboratoire Universitaire de Particules et Matières (LUPM), Université de Montpellier (34293)

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# 10 commandments for Hubble hunters

- 1 I am  $H_0 \approx 74$  thy Goal
- 2 Thou shalt not fail to fit key data (BAO, SNela, polarization)...
- 3 ...or include a local  $H_0$  prior in vain
- 4 Remember to not just blow up the uncertainty on  $H_0$ ...
- 5 ...honour its central value, and keep an eye on your  $\Delta\chi^2$ /Bayesian evidence
- 6 Thou shalt not murder  $\sigma_8/S_8$ ...
- 7 ...but aim to solve this and other tensions/puzzles at the same time
- 8 Thy solution shall come from a compelling particle/gravity model...
- 9 ...which makes verifiable predictions...
- 10 ...which later better be verified!



Credits: Gustave Doré

# Efficient model independent requirements to solve the tensions

- In general, to avoid the  $H_0$  tension one needs a positive correction to the first Friedmann equation at late times that could yield an increase in  $H_0$  compared to the  $\Lambda$ CDM scenario.

# Efficient model independent requirements to solve the tensions

- For the  $\sigma_8$  tension, we recall that in any cosmological model, at sub-Hubble scales and through matter epoch, the equation that governs the evolution of matter perturbations in the linear regime is

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\text{eff}}\rho_m\delta, \quad (1)$$

where  $G_{\text{eff}}$  is the effective gravitational coupling given by a generalized Poisson equation.

- Solving for  $\delta(a)$  provides the observable quantity  $f\sigma_8(a)$ , following the definitions  $f(a) \equiv d \ln \delta(a) / d \ln a$  and  $\sigma(a) = \sigma_8 \delta(1) / \delta(a=1)$ . Hence, alleviation of the  $\sigma_8$  tension may be obtained if  $G_{\text{eff}}$  becomes smaller than  $G_N$  during the growth of matter perturbations and/or if the “friction” term in (1) increases.

# Efficient model independent requirements to solve the tensions

We consider a correction in the first Friedmann equation of the form

$$H(z) = -\frac{d(z)}{4} + \sqrt{\frac{d^2(z)}{16} + H_{\Lambda\text{CDM}}^2(z)}, \quad (2)$$

where  $H_{\Lambda\text{CDM}}(z) \equiv H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$  is the Hubble rate in  $\Lambda\text{CDM}$ , with  $\Omega_m = \rho_m / (3M_\rho^2 H^2)$  the matter density parameter and primes denote derivatives with respect to  $z$ .

- If  $d < 0$  and is suitably chosen, one can have  $H(z \rightarrow z_{\text{CMB}}) \approx H_{\Lambda\text{CDM}}(z \rightarrow z_{\text{CMB}})$  but  $H(z \rightarrow 0) > H_{\Lambda\text{CDM}}(z \rightarrow 0)$ ; i.e., the  $H_0$  tension is solved [one should choose  $|d(z)| < H(z)$ , and thus, since  $H(z)$  decreases for smaller  $z$ , the deviation from  $\Lambda\text{CDM}$  will be significant only at low redshift].
- Since the friction term in (1) increases, the growth of structure gets damped, and therefore, the  $\sigma_8$  tension is also solved.

# General Relativity

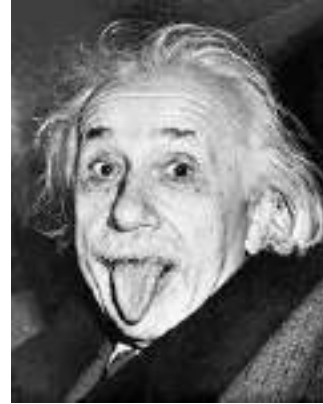
## Assumptions and Considerations

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] + \int d^4x L_m(g_{\mu\nu}, \psi)$$

- Diffeomorphism invariance
- Spacetime dimensionality=4
- **Geometry=Curvature** (connection=Levi Civita)
- Linear in Ricci scalar
- **Metric compatibility** (zero non-metricity)
- Minimal matter coupling
- Equivalence principle
- Lorentz invariance
- Locality

# Standard Model vs General Relativity Lagrangians

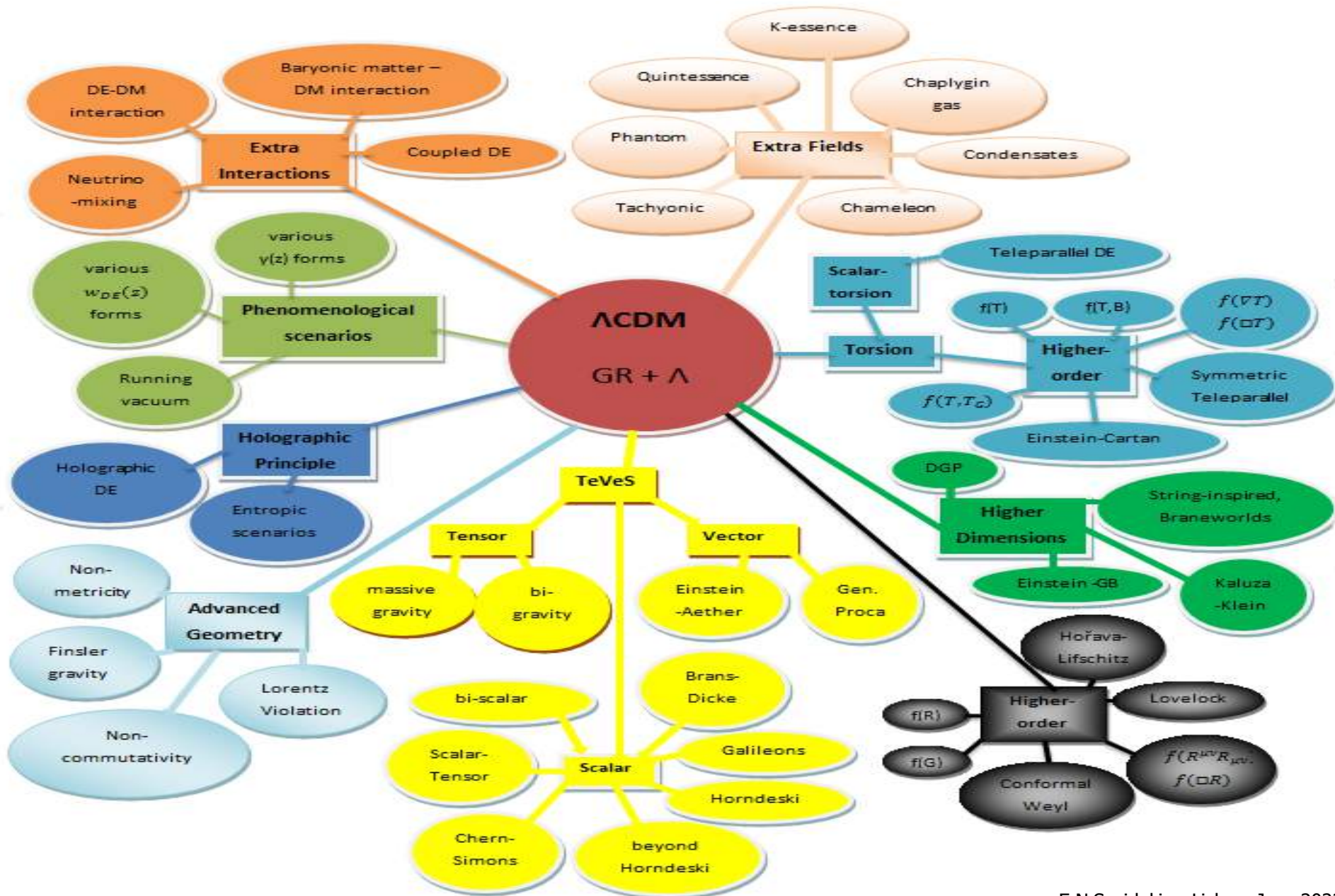
$$S = -\frac{1}{16\pi G} \int \sqrt{-g}(R(g)+2\Lambda) d^4x$$



$$\begin{aligned}
 & -\frac{1}{2}\partial_\mu g_\nu^\alpha \partial_\nu g_\mu^\alpha - g_\nu^\alpha f^{\beta\alpha\gamma} \partial_\mu g_\beta^\gamma g_\mu^\alpha g_\nu^\alpha - \frac{1}{4}g_\nu^\alpha f^{\beta\alpha\gamma} f^{\beta\alpha\delta} g_\mu^\gamma g_\mu^\delta g_\nu^\alpha + \\
 & \frac{1}{2}ig_\nu^\alpha (\partial_\mu^\beta \gamma^\alpha \partial_\mu^\beta) g_\mu^\alpha + G^\alpha \partial^\alpha G^\alpha + g_\nu^\alpha f^{\beta\alpha\gamma} \partial_\mu G^\beta G^\gamma g_\mu^\alpha - \partial_\mu W_\nu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\nu A_\mu \partial_\nu A_\mu - \frac{1}{2}\partial_\nu H \partial_\nu H - \\
 & \frac{1}{2}m_\nu^2 \bar{\psi} \psi - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\nu \phi^0 \partial_\nu \phi^0 - \frac{1}{2}M\phi^0 \phi^0 - \partial_\mu \left[ \frac{2M}{g} \phi^+ + \right. \\
 & \left. \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M^2}{g} \alpha_\lambda - ig_{\nu\alpha} [\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\nu^+) + Z_\mu^0 (W_\nu^- \partial_\nu W_\mu^+ - \\
 & W_\mu^+ \partial_\nu W_\nu^-)] - ig_{\nu\alpha} [\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\mu (W_\nu^+ \partial_\nu W_\mu^- - \\
 & W_\mu^- \partial_\nu W_\nu^+) + A_\mu (W_\nu^- \partial_\nu W_\mu^+ - W_\mu^+ \partial_\nu W_\nu^-)] - \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + \\
 & \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\nu^+ Z_\mu^0 W_\nu^- - Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + \\
 & g^2 s_w^2 (A_\mu W_\nu^+ A_\mu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\nu^0 W_\mu^+ W_\nu^-] - g\alpha [H^2 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
 & \frac{1}{2}g^2 \alpha_\lambda [H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
 & g M W_\mu^+ W_\nu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\nu^0 H - \frac{1}{2}ig [W_\mu^+ (\phi^0 \partial_\nu \phi^- - \phi^- \partial_\nu \phi^0) - \\
 & W_\nu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g [W_\mu^+ (H \partial_\nu \phi^- - \phi^- \partial_\nu H) - W_\nu^- (H \partial_\mu \phi^+ - \\
 & \phi^+ \partial_\mu H)] + \frac{1}{2}g \frac{1}{c_w} [Z_\mu^0 (H \partial_\nu \phi^0 - \phi^0 \partial_\nu H) - ig \frac{2M}{c_w} Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \\
 & ig_{\nu\alpha} M A_\mu (W_\nu^+ \phi^- - W_\nu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\nu \phi^- - \phi^- \partial_\nu \phi^+) + \\
 & ig_{\nu\alpha} A_\mu (\phi^+ \partial_\nu \phi^- - \phi^- \partial_\nu \phi^+) - \frac{1}{2}g^2 W_\mu^+ W_\nu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
 & \frac{1}{2}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\nu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \frac{1}{2}g^2 \frac{2M}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + \\
 & W_\nu^- \phi^+) - \frac{1}{2}ig^2 \frac{2M}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\nu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\nu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\nu^- \phi^+) - g^2 \frac{2M}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\nu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\nu \phi^+ \phi^- - e^2 (\gamma \partial + m_e^2) e^\lambda - e^2 \gamma \partial e^\lambda - \bar{u}_j^2 (\gamma \partial + m_u^2) u_j^2 - \\
 & d_j^2 (\gamma \partial + m_d^2) d_j^2 + ig_{\nu\alpha} A_\nu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^2 \gamma^\mu u_j^2) - \frac{1}{3}(\bar{d}_j^2 \gamma^\mu d_j^2)] + \\
 & \frac{19}{12} Z_\mu^0 [(\bar{\rho}^\lambda \gamma^\mu (1 + \gamma^5) \rho^\lambda) + (e^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^2 \gamma^\mu (\frac{4}{3}s_w^2 - \\
 & 1 - \gamma^5) u_j^2) + (\bar{d}_j^2 \gamma^\mu (1 - \frac{2}{3}s_w^2 - \gamma^5) d_j^2)] + \frac{19}{2\sqrt{2}} W_\mu^+ [(\bar{\rho}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + \\
 & (\bar{u}_j^2 \gamma^\mu (1 + \gamma^5) C_{3u} d_j^2) + \frac{19}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \rho^\lambda) + (\bar{d}_j^2 C_{3d}^1 \gamma^\mu (1 + \\
 & \gamma^5) u_j^2)] + \frac{19}{2\sqrt{2}} \frac{m_e^2}{M} [-\phi^+ (e^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (e^\lambda (1 + \gamma^5) e^\lambda)] - \\
 & \frac{g}{2} \frac{m_e^2}{M} [H(e^\lambda e^\lambda) + i\phi^0 (e^\lambda \gamma^5 e^\lambda)] + \frac{19}{23M\sqrt{2}} \phi^+ [-m_e^2 (\bar{u}_j^2 C_{3u} (1 - \gamma^5) d_j^2) + \\
 & m_e^2 (\bar{u}_j^2 C_{3u} (1 + \gamma^5) d_j^2) + \frac{19}{23M\sqrt{2}} \phi^- [m_e^2 (\bar{d}_j^2 C_{3d}^1 (1 + \gamma^5) u_j^2) - m_e^2 (\bar{d}_j^2 C_{3d}^1 (1 - \\
 & \gamma^5) u_j^2) - \frac{g}{2} \frac{m_e^2}{M} H(\bar{u}_j^2 u_j^2) - \frac{g}{2} \frac{m_e^2}{M} H(\bar{d}_j^2 d_j^2) + \frac{19}{2} \frac{m_e^2}{M} \phi^0 (\bar{u}_j^2 \gamma^5 u_j^2) - \\
 & \frac{19}{2} \frac{m_e^2}{M} \phi^0 (\bar{d}_j^2 \gamma^5 d_j^2) + \bar{X}^+ (\partial^\mu - M^2) X^+ + \bar{X}^- (\partial^\mu - M^2) X^- + X^0 (\partial^\mu - \\
 & \frac{M^2}{c_w^2}) X^0 + Y \partial^\mu Y + ig_{\nu\alpha} W_\mu^+ (\partial_\nu \bar{X}^0 X^- - \partial_\nu \bar{X}^+ X^0) + ig_{\nu\alpha} W_\mu^- (\partial_\nu \bar{X}^- X^0 - \\
 & \partial_\nu \bar{X}^+ X^-) + ig_{\nu\alpha} W_\mu^+ (\partial_\nu \bar{X}^- X^0 - \partial_\nu \bar{X}^+ X^-) + ig_{\nu\alpha} W_\mu^- (\partial_\nu \bar{X}^+ X^0 - \\
 & \partial_\nu \bar{X}^- X^-) + ig_{\nu\alpha} Z_\mu^0 (\partial_\nu \bar{X}^+ X^+ - \partial_\nu \bar{X}^- X^-) + ig_{\nu\alpha} A_\mu (\partial_\nu \bar{X}^+ X^+ - \\
 & \partial_\nu \bar{X}^- X^-) - \frac{1}{2}g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{2} \bar{X}^0 X^0 H] + \\
 & \frac{1-2c_w^2}{2c_w} ig M [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{c_w} ig M [\bar{X}^0 X^- \phi^+ - \bar{X}^+ X^+ \phi^-] + \\
 & ig M s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^+ X^+ \phi^-] + \frac{3}{2} ig M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
 \end{aligned}$$



# Modified Gravity



“Those that do not know geometry are not allowed to enter”.  
Front Door of Plato’s Academy



## Teleparallel Equivalent of General Relativity (TEGR)

In torsional formulation we use the vierbeins fields  $\mathbf{e}_A(x^\mu)$  as dynamical variables, which at a manifold point  $x^\mu$  form an orthonormal basis ( $\mathbf{e}_A \cdot \mathbf{e}_B = \eta_{AB}$  with  $\eta_{AB} = \text{diag}(1, -1, -1, -1)$ ).

In a coordinate basis they read as  $\mathbf{e}_A = e_A^\mu \partial_\mu$  and the metric is given by

$$g_{\mu\nu}(x) = \eta_{AB} e_\mu^A(x) e_\nu^B(x),$$

with Greek and Latin indices used for the coordinate and tangent space respectively.

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

# Teleparallel Equivalent of General Relativity (TEGR)

- Concerning the connection one introduces the Weitzenböck one, namely  $\overset{\mathbf{w}}{\Gamma}_{\nu\mu}^{\lambda} \equiv e_A^{\lambda} \partial_{\mu} e_{\nu}^A$ , and thus the corresponding torsion tensor becomes

$$T_{\mu\nu}^{\lambda} \equiv \overset{\mathbf{w}}{\Gamma}_{\nu\mu}^{\lambda} - \overset{\mathbf{w}}{\Gamma}_{\mu\nu}^{\lambda} = e_A^{\lambda} (\partial_{\mu} e_{\nu}^A - \partial_{\nu} e_{\mu}^A).$$

- The torsion tensor contains all information of the gravitational field, and its contraction provides the torsion scalar

$$T \equiv \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\rho} T^{\nu\mu}_{\nu},$$

which forms the Lagrangian of teleparallel gravity (in similar lines to the fact that the Ricci scalar forms the Lagrangian of general relativity).

# f(T) Gravity and f(T) Cosmology

- One can use TEGR as the starting point of gravitational modifications. The simplest direction is to generalize  $T$  to a function  $T + f(T)$  in the action:

$$S = \frac{1}{16\pi G} \int d^4x e [T + f(T) + L_m],$$

- Hence, we extract the Friedmann equations for  $f(T)$  cosmology as

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{f}{6} + \frac{Tf_T}{3}$$
$$\dot{H} = -\frac{4\pi G(\rho_m + P_m + \rho_r + P_r)}{1 + f_T + 2Tf_{TT}},$$

# Solving $H_0$ and $S_8$ tensions in $f(T)$ Gravity

- We consider the following ansatz:

$$f(T) = -[T + 6H_0^2(1 - \Omega_{m0}) + F(T)], \quad (9)$$

where  $F(T)$  describes the deviation from GR

The first Friedmann equation becomes

$$T(z) + 2\frac{F'(z)}{T'(z)}T(z) - F(z) = 6H_{\Lambda\text{CDM}}^2(z). \quad (10)$$

- In order to solve the  $H_0$  tension, we need  $T(0) = 6H_0^2 \simeq 6(H_0^{\text{CC}})^2$ , with  $H_0^{\text{CC}} = 74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , while in the early era of  $z \gtrsim 1100$  we require the Universe expansion to evolve as in  $\Lambda\text{CDM}$ , namely  $H(z \gtrsim 1100) \simeq H_{\Lambda\text{CDM}}(z \gtrsim 1100)$ . This implies  $F(z)|_{z \gtrsim 1100} \simeq cT^{1/2}(z)$  (the value  $c = 0$  corresponds to standard GR, while for  $c \neq 0$  we obtain  $\Lambda\text{CDM}$  too).

# Solving H0 and S8 tensions in f(T) Gravity

The effective gravitational coupling is given by

$$G_{\text{eff}} = \frac{G_N}{1 + F_T}. \quad (11)$$

Therefore, the perturbation equation becomes

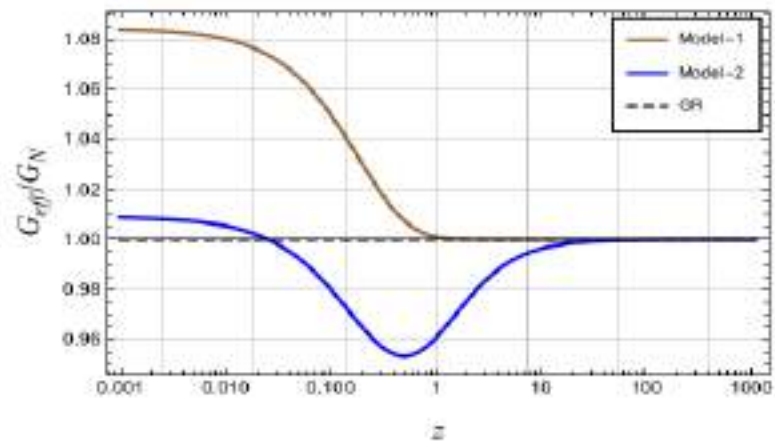
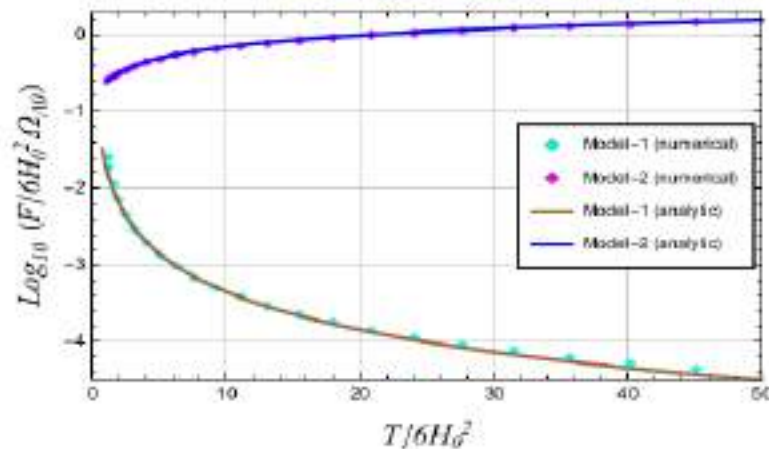
$$\delta'' + \left[ \frac{T'(z)}{2T(z)} - \frac{1}{1+z} \right] \delta' = \frac{9H_0^2 \Omega_{m0} (1+z)}{[1 + F'(z)/T'(z)]T(z)} \delta. \quad (12)$$

Since around the last scattering moment  $z \gtrsim 1100$  the Universe should be matter-dominated, we impose

$\delta'(z)|_{z \gtrsim 1100} \simeq -\frac{1}{1+z} \delta(z)$ , while at late times we look for  $\delta(z)$  that leads to an  $f\sigma_8$  in agreement with redshift survey observations.

# Solving H0 and S8 tensions in f(T) Gravity

By solving (10) and (12) with initial and boundary conditions at  $z \sim 0$  and  $z \sim 1100$ , we can find the functional forms for the free functions of the  $f(T)$  gravity that we consider, namely,  $T(z)$  and  $F(z)$ , that can alleviate both  $H_0$  and  $\sigma_8$  tensions.



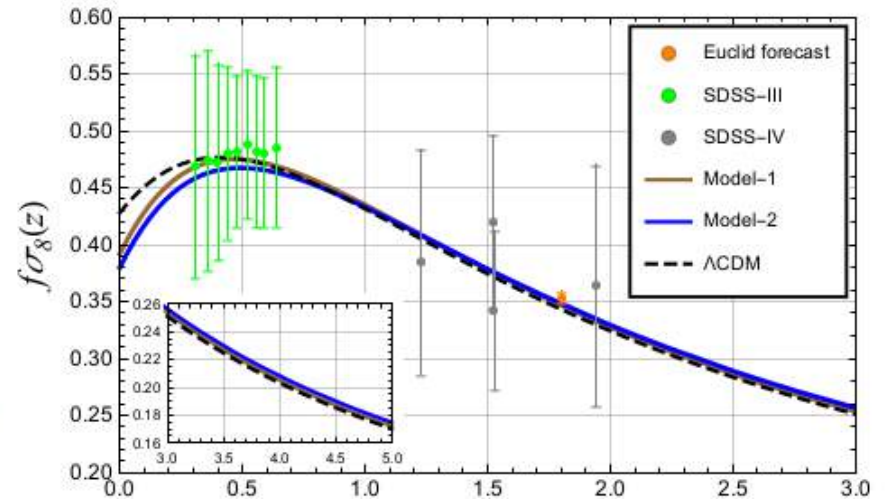
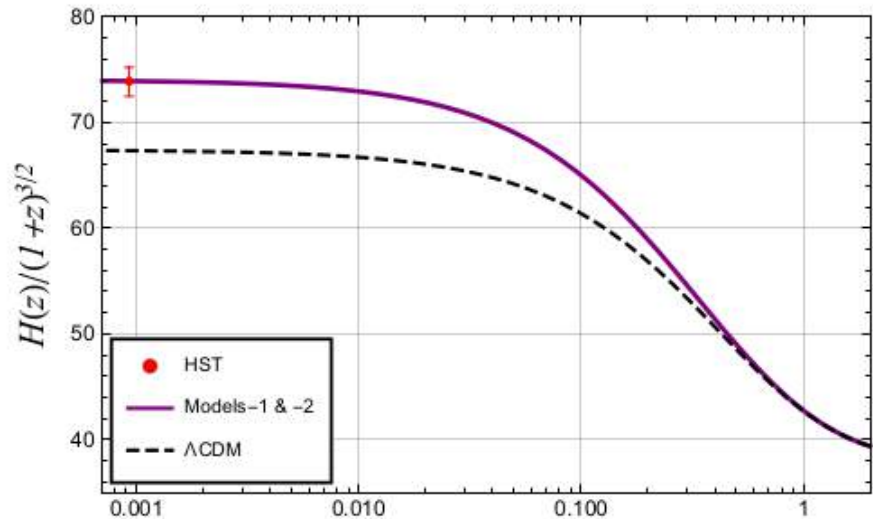
$$\text{Model-1: } F(T) \approx 375.47 \left( \frac{T}{6H_0^2} \right)^{-1.65}$$

$$\text{Model-2: } F(T) \approx 375.47 \left( \frac{T}{6H_0^2} \right)^{-1.65} + 25T^{1/2}.$$

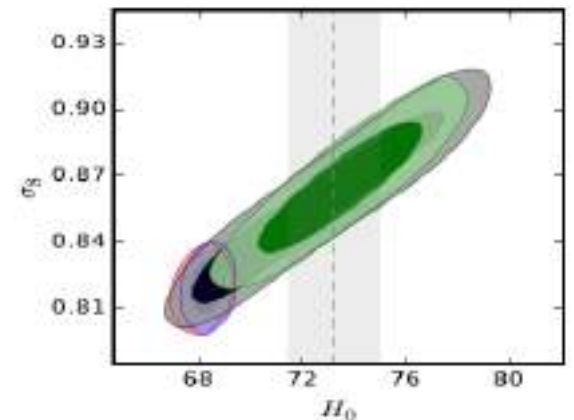
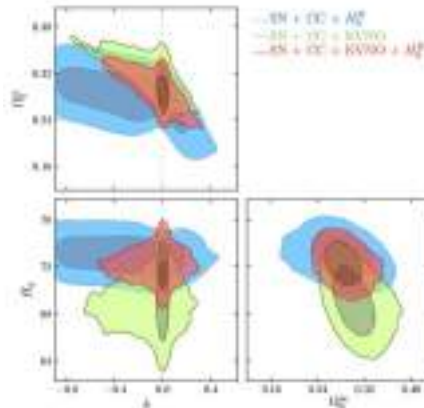
[S-F Yan, P. Zhang, J\_W Chen, X\_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]



# Solving H0 and S8 tensions in f(T) Gravity



Parameter	CMB + BAO	CMB + BAO + $H_0$
$10^2 \omega_b$	$2.235^{+0.013}_{-0.013}$	$2.235^{+0.013}_{-0.013}$
$\omega_{cdm}$	$0.1181^{+0.001}_{-0.001}$	$0.118^{+0.001}_{-0.001}$
$100\theta_s$	$1.041^{+0.0027}_{-0.0027}$	$1.041^{+0.0029}_{-0.0027}$
$\ln 10^{10} A_s$	$3.078^{+0.023}_{-0.023}$	$3.08^{+0.022}_{-0.022}$
$w_s$	$0.9678^{+0.0039}_{-0.0039}$	$0.9684^{+0.0039}_{-0.0044}$
$\tau_{reio}$	$0.073^{+0.012}_{-0.013}$	$0.075^{+0.012}_{-0.012}$
$n$	$0.0043^{+0.0033}_{-0.0033}$	$0.0054^{+0.0030}_{-0.0030}$
$\log \alpha$	$10.00^{+0.081}_{-0.12}$	$10.03^{+0.06}_{-0.06}$
$\Omega_{FD}$	$0.73^{+0.021}_{-0.029}$	$0.738^{+0.015}_{-0.015}$
$H_0$	$72.4^{+3.3}_{-4.1}$	$73.5^{+2.1}_{-2.1}$
$\sigma_8$	$0.855^{+0.023}_{-0.023}$	$0.866^{+0.02}_{-0.02}$
$\chi^2_{min}/2$	6480.48	6482.27

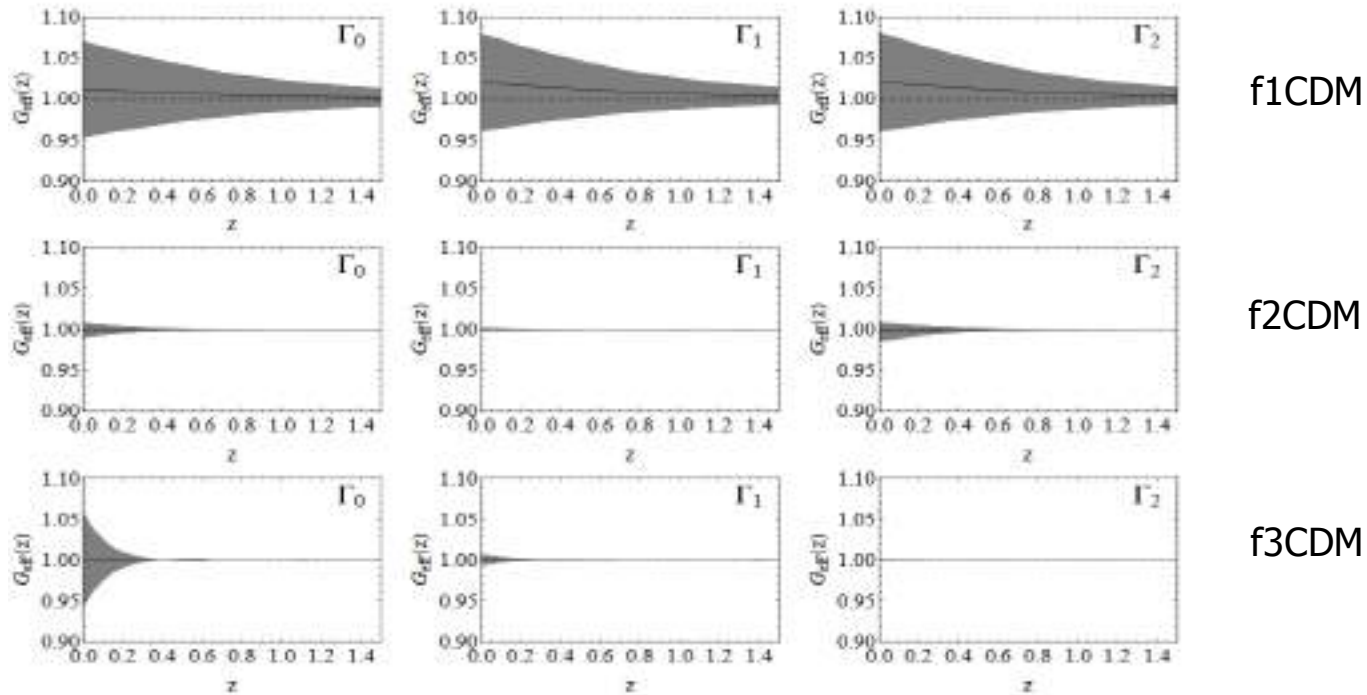


[S-F Yan, P. Zhang, J-W Chen, X\_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]

[J-W Chen, W. Luo, Y-F Cai, E.N. Saridakis, PRD 102]

[S. Basilakos, S. Nesseris, F. Anagnostopoulos, E.N.Saridakis, JCAP 2019]

# Viabale $f(T)$ models



- In  **$f(T)$  gravity** we can indeed obtain  $G_{\text{eff}}/G_{\text{N}} < 1$  for  $z < 2$ , without affecting the background evolution.
- **$f\sigma_8$  tension** may be **alleviated**. [Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

# In other modified gravities: Not possible

- This behavior **is not possible** in other **modified gravities**. e.g.:

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} f(R, \phi, X) + \mathcal{L}_m \right) \quad X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$G_{\text{eff}}(a, k)/G_{\text{N}} = \frac{1}{F} \frac{f_{,X} + 4 \left( f_{,X} \frac{k^2}{a^2} \frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F} \right)}{f_{,X} + 3 \left( f_{,X} \frac{k^2}{a^2} \frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F} \right)} \quad F = F(R, \phi, X) = \partial_R f(R, \phi, X)$$

- $G_{\text{eff}}/G_{\text{N}} > 1$  for all models that **do not have ghosts** (i.e. with  $f_R, f_{RR} > 0$ ).
- On the contrary, **f(T) gravity** has **second-order field equations** and moreover **perturbations are stable** in a large part of the parameter phase.

# Solving H0 and S8 tensions in f(T) Gravity

- We conclude that the class of  $f(T)$  gravity:  
 $f(T) = -T - 2\Lambda/M_p^2 + \alpha T^\beta$ , where only two out of the three parameters  $\Lambda$ ,  $\alpha$ , and  $\beta$  are independent (the third one is eliminated using  $\Omega_{m0}$ ), can alleviate both  $H_0$  and  $\sigma_8$  tensions with suitable parameter choices.
- Such kinds of models in  $f(T)$  gravity could also be examined through galaxy-galaxy lensing effects [Z. Chen, W. Luo, Y.F. Cai and E.N. Saridakis, Phys.Rev.D 102 (2020) 10, 104044], strong lensing effects around black holes [S. Yan et. al, Phys.Rev.Res. 2 (2020) 2, 023164] and gravitational wave experiments [Y.F. Cai, C. Li, E.N. Saridakis and L. Xue, Phys. Rev. D 97, no. 10, 103513 (2018)].

# Conclusions

- i) **Astrophysics** and **Cosmology** have become **precision** sciences.
- ii) A **huge amount of accumulating data** suggest possible **tensions** with theoretical predictions of  $\Lambda$ CDM paradigm.
- iii) **New Physics** or **paradigm shift** may be the **way out**
- iv) We can **modify** the **Universe content**, the **interactions**, or/and the **gravitational theory**.



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**THANK YOU!**