Cosmological implications of Weyl geometric gravity

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Content

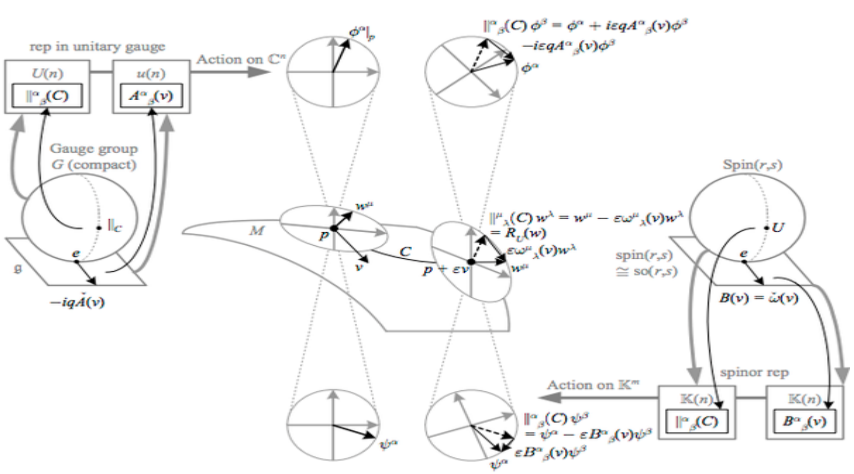
Basics of Weyl geometry

Hermann Weyl (1885-1955

In 1918 Hermann Weyl introduced an extension of Riemann geometry with the main physical goal of unifying geometrically gravity and electromagnetism

Einstein strongly criticized Weyl geometry as a physical theory

In 1929 Weyl showed that electrodynamics is invariant under the gauge transformations of the gauge field and the wave function of the charged field



Gauge theory, fundamental for particle physics, was born from Weyl geometry

Basics of Weyl geometry

Weyl geometry: classes of equivalence $(g_{\alpha\beta}, \omega_\mu)$ of the metric $g_{\alpha\beta}$ and of the vector gauge field ω_μ, related by the gauge transformations,

$$\tilde{g}_{\mu\nu} = \Sigma^n g_{\mu\nu} = \left[\tilde{g}_{\mu\nu}\right], \tilde{\omega}_\mu = \omega_\mu - \frac{1}{\alpha}\partial_\mu \ln \Sigma$$

$$\sqrt{-\tilde{g}} = \Sigma^{2n}\sqrt{-g}, \tilde{\phi} = \Sigma^{-n/2}\phi.$$

Weyl connection: Weyl geometry is non-metric

$$\tilde{\nabla}_\lambda g_{\mu\nu} = -n\alpha\omega_\mu g_{\mu\nu}$$

$$\tilde{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + \alpha\frac{n}{2}\left(\delta^\lambda_\mu\omega_\nu + \delta^\lambda_\nu\omega_\mu - \omega^\lambda g_{\mu\nu}\right)$$

$$\Gamma_{\lambda,\mu\nu} = \frac{1}{2}\left(\partial_\nu g_{\lambda\mu} + \partial_\mu g_{\lambda\nu} - \partial_\lambda g_{\mu\nu}\right)$$

Basics of Weyl geometry

- Strength of the Weyl vector
- $$\tilde{F}_{\mu\nu} = \nabla_\mu \omega_\nu - \nabla_\nu \omega_\mu$$
- Curvature tensor

$$\tilde{R}^{\lambda}_{\mu\nu\sigma} = \partial_\nu \tilde{\Gamma}^{\lambda}_{\mu\sigma} - \partial_\sigma \tilde{\Gamma}^{\lambda}_{\mu\nu} + \tilde{\Gamma}^{\lambda}_{\rho\nu} \tilde{\Gamma}^{\rho}_{\mu\sigma} - \tilde{\Gamma}^{\lambda}_{\rho\sigma} \tilde{\Gamma}^{\rho}_{\mu\nu}$$

Weyl scalar

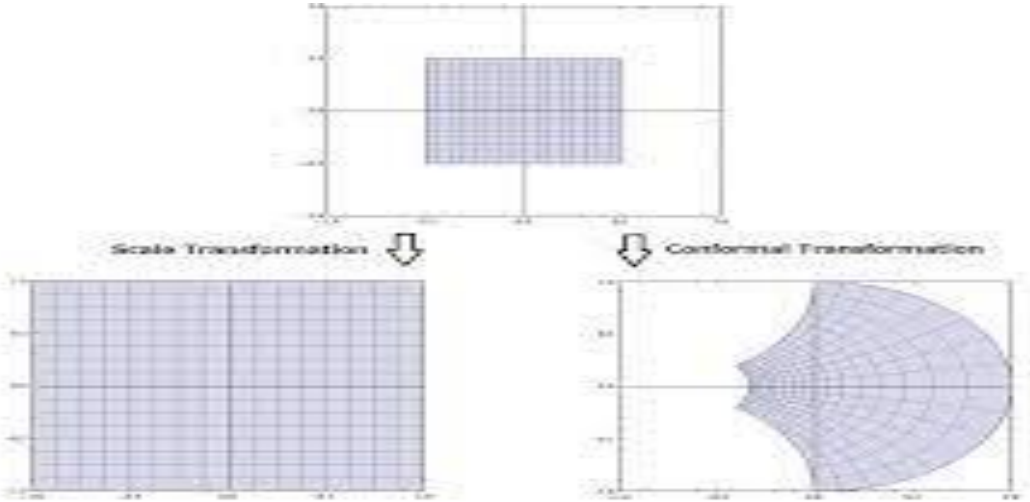
$$\tilde{R} = R - 3n\alpha \nabla_\mu \omega^\mu - \frac{3}{2}(n\alpha)^2 \omega_\mu \omega^\mu$$

Weyl tensor

$$\tilde{C}_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} - \frac{n\alpha}{4}\Big(g_{\mu\rho}\tilde{F}_{\nu\sigma} + g_{\nu\sigma}\tilde{F}_{\mu\rho} - g_{\mu\sigma}\tilde{F}_{\nu\rho}$$
$$-g_{\nu\rho}\tilde{F}_{\mu\sigma}\Big) - \frac{\alpha n}{2}\tilde{F}_{\mu\nu} g_{\rho\sigma}.$$

Conformal coupling of matter and geometry

- Conformal transformations: stretching all lengths (due to change of units) by factors that depend only on the spacetime locations



Conformal invariance: initially discussed by Weyl

Highly attractive idea, similar to the gauge principle that enriched so much contemporary physics

Global units transformations are analogous to global gauge transformations or global internal-symmetry transformations

Conformal coupling of matter and geometry

The extension of units transformations to the local level, and the requirement of conformal invariance of physical laws is similar to the promotion of gauge and internal invariances to the local level by the introduction of gauge fields

- Maxwell's equations, the massless Dirac equation, the massless scalar field equations, the electromagnetic, weak, and strong interactions between elementary particle fields are all conformally invariant

Microscopic physics is conformally invariant in its entirety

But Einstein gravity is not!

Conformal coupling of matter in Weyl geometry

Conformally invariant Einstein gravity: (D. M. Ghilencea, JHEP **03** 049 (2019); D. M. Ghilencea, Phys. Rev. **D 101**, 045010 (2020) D. M. Ghilencea, Eur. Phys. J. **C 80**, 1147 (2020); Eur. Phys. J. **C 81**, 510 (2021); arXiv:2104.15118 (2021).)

$$S_0 = \int \left[\frac{1}{4!}\frac{1}{\xi^2}\tilde{R}^2 - \frac{1}{4}\tilde{F}^2_{\mu\nu} - \frac{1}{\eta^2}\tilde{C}^2_{\mu\nu\rho\sigma}\right]\sqrt{-g}d^4x, \quad (26)$$

The Weyl action has spontaneous symmetry breaking in a Stueckelberg mechanism

The Weyl gauge field acquires mass

One recovers the Einstein-Hilbert action of standard general relativity with a positive cosmological constant,

One obtains the Proca action for the massive Weyl gauge field

Conformal coupling of matter in Weyl geometry

- Conformally invariant coupling of matter to curvature in Weyl geometry: conformal f(R, Lm) theory (Harko and Shahidi, EPJC 82, 219 (2022); Harko and Shahidi, arXiv:2210.03631 (2022))

$$S=\int\left[\frac{1}{4!\xi^2}\tilde{R}^2-\frac{1}{4}\tilde{F}^2_{\mu\nu}-\frac{1}{\eta^2}\tilde{C}^2_{\mu\nu\rho\sigma}+\frac{1}{4!\gamma^2}L_m\tilde{R}^2\right]\sqrt{-g}d^4x$$
$$=\int\left[\frac{1}{4!\xi^2}\left(1+\frac{\xi^2}{\gamma^2}L_m\right)\tilde{R}^2-\frac{1}{4}\tilde{F}^2_{\mu\nu}-\frac{1}{\eta^2}\tilde{C}^2_{\mu\nu\rho\sigma}\right]\sqrt{-g}d^4x.$$

L_m matter Lagrangian ξ,η,γ coupling constants

Conformal coupling of matter in Weyl geometry

- Linear/scalar representation of quadratic Weyl gravity
- (D. M. Ghilencea, Eur. Phys. J. **C 80**, 1147 (2020); Eur. Phys. J. **C 81**, 510 (2021); arXiv:2104.15118 (2021))

$$\tilde{R}^2 \to -2\phi_0^2\tilde{R} - \phi_0^4$$

$$S = -\int \left\{ \frac{1}{2\xi^2}\left(1 + \frac{\xi^2}{\gamma^2}L_m\right)\left[\frac{\phi_0^2}{6}R - \frac{\alpha}{2}\phi_0^2\nabla_\mu\omega^\mu - \frac{\alpha^2}{4}\phi_0^2\omega_\mu\omega^\mu + \frac{\phi_0^4}{12}\right] + \frac{1}{4}\tilde{F}_{\mu\nu}^2 + \frac{1}{\eta^2}\tilde{C}_{\mu\nu\rho\sigma}^2 \right\}\sqrt{-g}d^4x. \qquad (31)$$

$$\Sigma = \phi_0^2/\langle\phi_0^2\rangle \qquad \mathcal{L}_m = 1 + \frac{\xi^2}{\gamma^2}L_m$$

$$M_p^2 = \langle\phi_0^2\rangle/6\xi^2 \qquad 1/\delta^2 = 1 + 6\alpha^2/\eta^2$$

$$\nabla\mu\ \omega^\mu = 0$$

Conformal coupling of matter in Weyl geometry

Action of conformally invariant f(R,Lm) gravity theory (T. Harko and S. Shahidi, EPJC 82, 219 (2022); T. Harko and S. Shahidi EPJC 82, 1003, 2022))

$$S = -\int \left\{ \mathcal{L}_m \left[\frac{1}{2} M_p^2 R - \frac{3\alpha^2}{4} M_p^2 \omega_\mu \omega^\mu - \frac{3}{2}\xi^2 M_p^4 \right] + \frac{1}{4\delta^2} \tilde{F}_{\mu\nu}^2 + \frac{1}{\eta^2} C_{\mu\nu\rho\sigma}^2 \right\} \sqrt{-g} d^4 x, \qquad (34)$$

The conformally invariant Weyl geometric gravitational action is defined in the Riemann space

Gravitational field equations

$$\nabla_\mu \tilde{F}^{\mu\nu} + \frac{3}{2} M_p^2 \alpha^2 \delta^2 \mathcal{L}_m \omega^\nu = 0$$

$$M_p^2 \left[\mathcal{L}_m R_{\mu\nu} + \left(g_{\mu\nu} \nabla_\alpha \nabla^\alpha - \nabla_\mu \nabla_\nu \right) \mathcal{L}_m - \frac{3\alpha^2}{2} \mathcal{L}_m \omega_\mu \omega_\nu \right]$$
$$- \frac{1}{2} M_p^2 \mathcal{T}_{\mu\nu} \left(R - \frac{3\alpha^2}{2} \omega_\alpha \omega_\beta g^{\alpha\beta} + 3\xi^2 M_p^2 \right)$$
$$+ \frac{8}{\eta^2} B_{\mu\nu} - 2\tilde{T}_{\mu\nu}^{(\omega)} = 0. \qquad (54)$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\partial\left(\sqrt{-g} L_m\right)}{\partial g^{\mu\nu}}$$

$$\mathcal{T}_{\mu\nu} = g_{\mu\nu} + \frac{\xi^2}{\gamma^2} T_{\mu\nu}$$

$$B_{\mu\nu} = \nabla_\lambda \nabla_\sigma C_{\mu\ \nu}^{\ \sigma\ \lambda} + \frac{1}{2} C_{\mu\ \nu}^{\ \lambda\ \sigma} R_{\lambda\sigma}$$

$$\tilde{T}_{\mu\nu}^{(\omega)} = \frac{1}{2\delta^2} \left(-\tilde{F}_{\mu\lambda} \tilde{F}_\nu^{\ \lambda} + \frac{1}{4} \tilde{F}_{\lambda\sigma} \tilde{F}^{\lambda\sigma} g_{\mu\nu} \right)$$

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu},$$

Gravitational field equations

- The trace equation

$$\left(\mathcal{L}_m R + 3\nabla_\alpha \nabla^\alpha \mathcal{L}_m - \frac{3\alpha^2}{2}\mathcal{L}_m \omega^2\right)$$
$$-\frac{1}{2}\mathcal{T}\left(R - \frac{3\alpha^2}{2}\omega^2 + 3\xi^2 M_p^2\right) = 0$$

Alternative form of the field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{8}{\eta^2 M_p^2 \mathcal{L}_m}B_{\mu\nu} + \frac{1}{\mathcal{L}_m}\hat{\Sigma}_{\mu\nu}\mathcal{L}_m$$
$$+\frac{1}{2}\left(g_{\mu\nu} - \frac{\mathcal{T}_{\mu\nu}}{\mathcal{L}_m}\right)R = -\frac{3}{2}\frac{1}{\mathcal{L}_m}\left(\frac{\alpha^2}{2}\omega^2 + \xi^2 M_p^2\right)\mathcal{T}_{\mu\nu}$$
$$+\frac{3\alpha^2}{2}\omega_\mu\omega_\nu + \frac{2}{M_p^2}\frac{1}{\mathcal{L}_m}\tilde{T}^{(\omega)}_{\mu\nu},$$

Evolution of the Weyl vector

$$\nabla^2 \omega^\nu + R^\nu_\beta \omega^\beta - \nabla^\nu\left(\nabla_\mu \omega^\mu\right) + \frac{3}{2}M_p^2 \alpha^2 \delta^2 \mathcal{L}_m \omega^\nu = 0. \quad (63)$$

Gravitational field equations

Divergence of the matter energy-momentum tensor

$$\nabla_\mu \mathcal{T}^\mu_\nu = \left(\mathcal{L}_m \delta^\mu_\nu - \mathcal{T}^\mu_\nu\right) \nabla_\mu \ln\left(R - \frac{3\alpha^2}{2}\omega^2 + 3\xi^2 M_p^2\right) - \frac{6\alpha^2 \omega_\nu \omega^\mu \nabla_\mu \mathcal{L}_m}{2R - 3\alpha^2\omega^2 + 6\xi^2 M_p^2} := Q_\nu .$$

Energy balance equation

$$\dot{\rho} + (\rho + p)\nabla_\mu u^\mu = \frac{\gamma^2}{\xi^2} u_\mu Q^\mu$$

Momentum balance equation

$$\frac{d^2 x^\lambda}{ds^2} + \Gamma^\lambda_{\mu\nu} u^\mu u^\nu = \frac{h^{\nu\lambda}}{\rho + p}\left(\frac{\gamma^2}{\xi^2} Q_\nu - \nabla_\nu p\right) = f^\lambda$$

Gravitational field equations

- Equations of the Weyl vector
- $\tilde{F}_{0k} = \partial_t \omega_k - \partial_k \omega_0 = \tilde{E}_k,\ k = 1,2,3$

$$\tilde{F}_{jk} = \partial_j \omega_k - \partial_k \omega_j = -\tilde{B}_{jk} = -\epsilon_{ijk}\tilde{B}^i, j, k = 1, 2, 3, \qquad (66)$$

$$\frac{\partial \vec{\tilde{B}}}{\partial t} + \nabla \times \vec{\tilde{E}} = 0, \nabla \cdot \vec{\tilde{B}} = 0$$

$$\partial_k \left(-g^{jj} g^{00} \sqrt{-g} \tilde{E}_j \right) + \frac{3}{2} M_p^2 \alpha^2 \delta^2 \mathcal{L}_m \omega^0 \sqrt{-g} = 0, \quad (69)$$

$$\epsilon_{ijk} \partial_j \left(g^{ii} g^{jj} \sqrt{-g} \tilde{B}^k \right) + \frac{3}{2} M_p^2 \alpha^2 \delta^2 \mathcal{L}_m \omega^k \sqrt{-g} = 0, \quad (70)$$

The Newtonian approximation

- The generalized Poisson equation

$$R^\mu_\nu = -\frac{1}{\mathcal{L}_m}\left(\delta^\mu_\nu \nabla_\alpha \nabla^\alpha - \nabla^\mu \nabla_\nu\right)\mathcal{L}_m + \frac{1}{2\mathcal{L}_m}\mathcal{T}^\mu_\nu\left(R - \frac{3\alpha^2}{2}\omega^2 + 3\xi^2 M_p^2\right) + \frac{3\alpha^2}{2}\omega^\mu\omega_\nu + \frac{2}{M_p^2\mathcal{L}_m}\tilde{T}^{(\omega)\mu}_\nu .$$

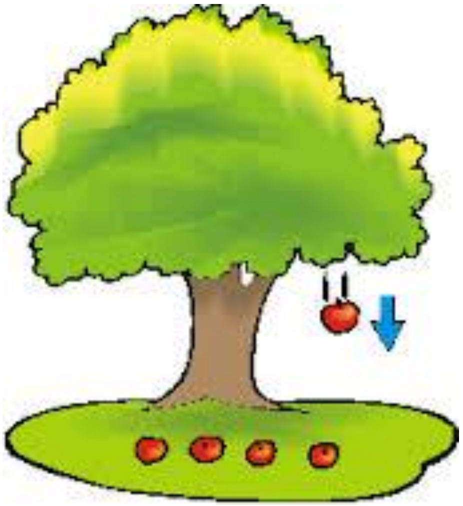
$T^0_0 = \rho$ $u^0 = u_0 = 1$, and $u^\alpha = 0$, $\alpha = 1,2,3$ $g_{00} = 1 + 2\varphi$

$$\left(1 + \frac{\xi^2}{\gamma^2}\rho\right)\Delta\varphi = \frac{3\xi^2}{\gamma^2}\left(\frac{\alpha^2}{2}\omega^2 + \xi^2 M_p^2\right)\rho + 6\left(\xi^2 M_p^2 - \frac{\alpha^2}{2}\omega^2\right)\varphi + \frac{2\xi^2}{\gamma^2}\Delta\rho + 3\left(\frac{\alpha^2}{2}\omega^2 + \xi^2 M_p^2\right). \quad (73)$$

$$\Delta\varphi = 6\left(\xi^2 M_p^2 - \frac{\alpha^2}{2}\omega^2\right)\varphi + 3\left(\frac{\alpha^2}{2}\omega^2 + \xi^2 M_p^2\right). \quad (74)$$

The Newtonian approximation

- Corrections to the vacuum Newtonian potential



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi(r)}{dr}\right)=\frac{3\alpha^2}{2}\omega^2(r)+3\xi^2M_p^2$$

$$\varphi(r)=-\frac{C}{r}+\frac{3\alpha^2}{2}\int^r d\varsigma\frac{1}{\varsigma^2}\int^{\varsigma}\theta^2\omega^2(\theta)\,d\theta+\frac{\xi^2M_p^2}{2}r^2. \tag{76}$$

$$\varphi(r)=-\frac{C}{r}+\frac{1}{2}\left(\frac{\alpha^2\omega^2}{2}+\xi^2M_p^2\right)r^2$$

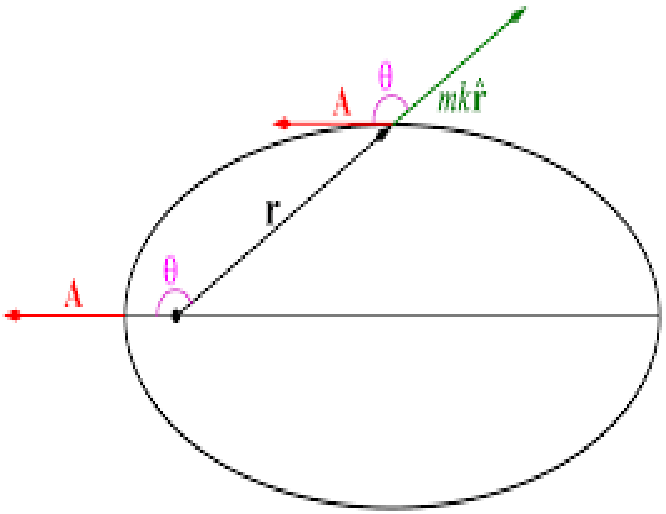
The modifications of the Newtonian potential could lead to some observational or experimental tests for the confirmation of the presence of Weyl geometry in the Universe.

The Newtonian approximation

Solar System test of quadratic Weyl gravity

-Weyl gravity can also be tested by investigating the orbital parameters of the motion of the planets around a central massive object (the Sun).

The motion of massive test particles in a gravitational field can be studied in a simple way with the help of the Runge-Lenz vector



$$\vec{A} = \vec{r} \times \vec{p} - mk\hat{r}$$

$$\left(\frac{L^2}{\mu\alpha_0}\right) r^{-1} = 1 + e\cos\theta.$$

$$\vec{A} = \left(\frac{\vec{L}^2}{\mu r} - \alpha_0\right)\vec{e}_r - \dot{r}L\vec{e}_\theta$$

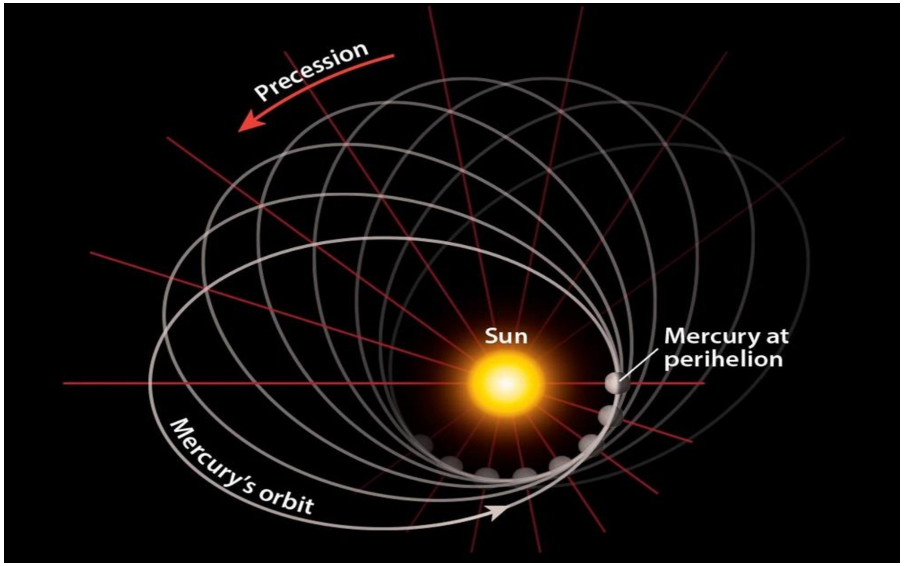
$$\frac{d\vec{A}}{d\theta} = r^2\left[\frac{dV(r)}{dr} - \frac{\alpha_0}{r^2}\right]\vec{e}_\theta$$

The Newtonian approximation

- Solar System test of quadratic Weyl gravity
- We model the gravitational field in the Solar System by a potential term consisting of two components:
- - the Post-Newtonian potential

$$V_{PN}(r) = -\frac{\alpha_0}{r} - 3\frac{\alpha_0^2}{mr^2}$$

-
- - extra contribution from Weyl geometry $V_W(r) = m\vec{a}_E$

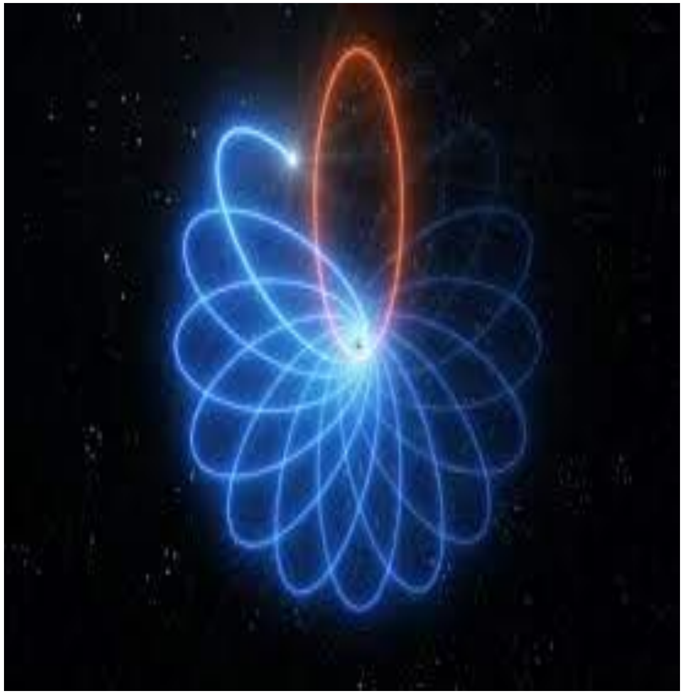


$$\frac{d\vec{A}}{d\theta} = r^2\left[6\frac{\alpha_0^2}{mr^3} + m\vec{a}_E(r)\right]\vec{e}_\theta$$

$$\Delta\tilde{\phi} = \frac{1}{\alpha_0 e}\int_0^{2\pi}\left|\vec{A}\times\frac{d\vec{A}}{d\theta}\right| d\theta.$$

The Newtonian approximation

- Solar System tests of quadratic Weyl gravity

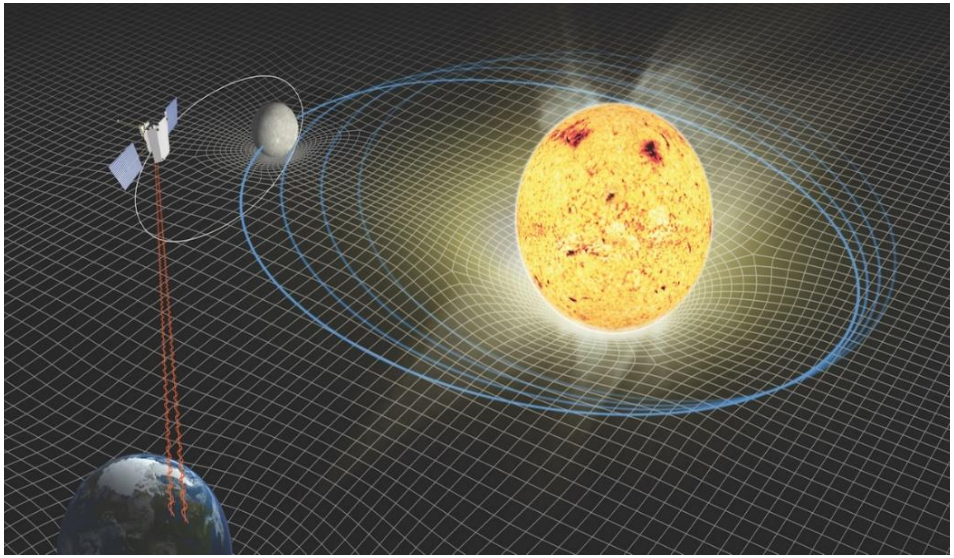


$$\Delta\tilde{\phi} = 24\pi^3 \left(\frac{a}{T}\right)^2 \frac{1}{1-e^2} + \frac{L}{8\pi^3 me} \frac{\left(1-e^2\right)^{3/2}}{\left(a/T\right)^3} \times \int_0^{2\pi} \frac{a_E \left[L^2 \left(1+e\cos\theta\right)^{-1}/m\alpha_0\right]}{\left(1+e\cos\theta\right)^2} \cos\theta d\theta, \qquad (105)$$

$$\Delta\tilde{\phi} = \frac{6\pi G M_\odot}{a\left(1-e^2\right)} + \frac{2\pi a^2\sqrt{1-e^2}}{G M_\odot} a_E$$

The Newtonian approximation

Solar System tests of quadratic Weyl gravity



$\left(\Delta\tilde{\phi}\right)_{obs}$ =43.11$\pm$0.21 arcsec/century

$\left(\Delta\tilde{\phi}\right)_{GR}$ =42.962 arcsec/century

$\left(\Delta\tilde{\phi}\right)_{W} = \left(\Delta\tilde{\phi}\right)_{obs} - \left(\Delta\tilde{\phi}\right)_{GR}$= 0.17 arcsec/century

Can be attributed to other physical effects

$$a_E < 1.28 \times 10^{-9}\ \text{cm}/s^2$$

This value does not rule out the possibility of the presence of Weyl geometric gravitational effects in the Solar System

The Newtonian approximation

- $F = -\lfloor \nabla \varphi(r) \rfloor = \frac{C}{r^2} - \frac{1}{2} r\left(\alpha^2 \omega^2 + \xi^2 M_p^2\right)$

$$a = a_N + a_E$$

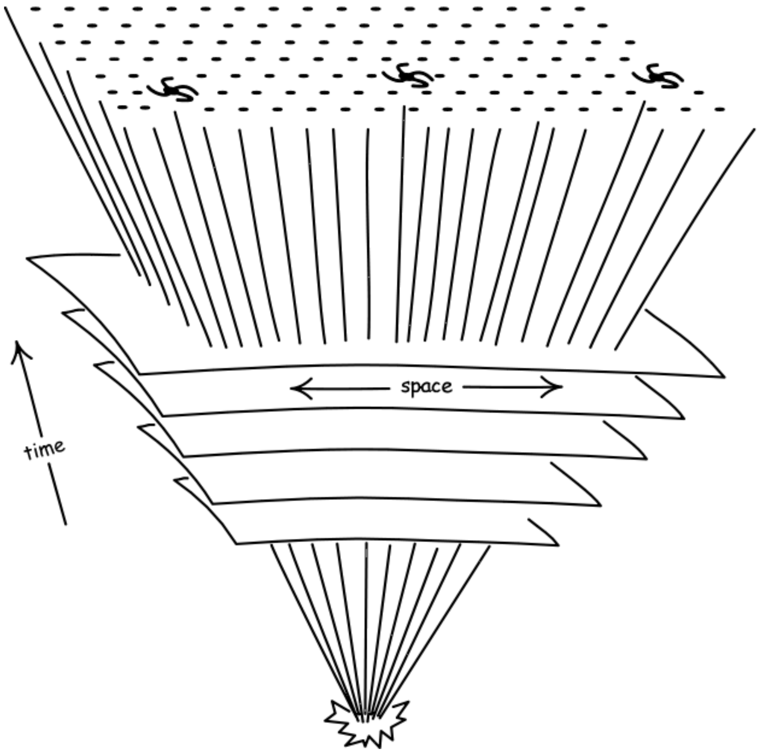
- $\left[\left(\alpha^2 \omega^2 + \xi^2 M_p^2\right)\right]_{Mercury} \leq 2.455\times 10^{-43} \frac{1}{cm^2}$

- Constraints on non-metricity

Delhom, Iarley P. Lobo, Olmo, Romero, Eur. Phys. J. C 80:415, (2020)

- Delhom-Latorre, Olmo, Ronco, Phys. Lett. B 780, 294 (2018)

Generalized Friedmann equations



Cosmological Friedmann-Lemaitre-Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

$$\omega_\mu = (\omega_0(t), 0,0, \omega_3(t))$$

$$T_\omega^\pm = \frac{\alpha^2 \phi_0^2}{8\,\xi^2} \frac{\omega_k\,\omega_k}{a^2} \pm \frac{\dot{\omega}_k \dot{\omega}_k}{2a^2}$$

Generalized Friedmann equations

Cosmological Weyl field equations

$$\frac{\partial}{\partial x^{\sigma}}\tilde{F}_{\mu\nu}+\frac{\partial}{\partial x^{\nu}}\tilde{F}_{\sigma\mu}+\frac{\partial}{\partial x^{\mu}}\tilde{F}_{\nu\sigma}=0$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}\tilde{F}^{\mu\nu}\right)+\frac{3}{2}M_p^2\alpha^2\delta^2\mathcal{L}_m\omega^{\nu}\sqrt{-g}=0, \qquad (147)$$

$$\tilde{F}_{\mu\nu}=a^2(\eta)\begin{pmatrix} 0 & \tilde{E}_1 & \tilde{E}_2 & \tilde{E}_3 \\ -\tilde{E}_1 & 0 & -\tilde{B}_3 & \tilde{B}_2 \\ -\tilde{E}_2 & \tilde{B}_3 & 0 & -\tilde{B}_1 \\ -\tilde{E}_3 & -\tilde{B}_2 & \tilde{B}_1 & 0 \end{pmatrix}$$

$$-\frac{1}{a^2}\frac{\partial}{\partial\eta}(a^2\vec{\tilde{B}})+\nabla\times\vec{\tilde{E}}=0 \qquad \nabla\cdot\vec{\tilde{B}}=0$$

$$\nabla\cdot\vec{\tilde{E}}-\frac{3}{2}M_p^2\alpha^2\delta^2a^2\mathcal{L}_m\omega_0=0$$

$$\nabla\times\vec{\tilde{B}}+\frac{1}{a^2}\frac{\partial}{\partial\eta}a^2\vec{\tilde{E}}+\frac{3}{2}M_p^2\alpha^2\delta^2a^2\mathcal{L}_m\vec{\omega}=0, \qquad (151)$$

Generalized Friedmann equations

Cosmological Weyl field equations

$$\langle X \rangle = \frac{1}{V_0} \lim_{V \to V_0} \int \sqrt{-g} X d^3 x^i$$

$$\omega_\mu = \left(a^2 \omega_0, a^2 \vec{\omega}\right)$$

$$\left\langle \tilde{E}_i \tilde{E}_j \right\rangle = \frac{1}{3} \left\langle \vec{\tilde{E}}^2 \right\rangle \delta_{ij}, \quad \left\langle \tilde{B}_i \tilde{B}_j \right\rangle = \frac{1}{3} \left\langle \vec{\tilde{B}}^2 \right\rangle \delta_{ij}, \quad (154)$$

$$\left\langle \tilde{T}_{ik}^{(\omega)} \right\rangle = \frac{1}{3} \left\langle \tilde{T}_{00}^{(w)} \right\rangle \delta_{ik}$$

The effects of the Weyl geometry can be modelled in terms of an effective fluid satisfying a radiation type equation of state

Generalized Friedmann equations

Field equations in the cosmological vacuum

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} - \frac{2\,\xi^2}{\phi_0^2} T_\omega^+ - \frac{\ddot{\phi}_0}{\phi_0} + 3\,H\,\frac{\dot{\phi}_0}{\phi_0} - \frac{\phi_0^2}{12} = 0$$

$$\frac{\dot{a}^2}{a^2} + 2\,\frac{\ddot{a}}{a} + \frac{\kappa}{a^2} + \frac{2\xi^2}{\phi_0^2} T_\omega^- + 3\frac{\ddot{\phi}_0}{\phi_0} + 9H\,\frac{\dot{\phi}_0}{\phi_0} - \frac{\phi_0^2}{4} = 0,$$

$$-\frac{\ddot{\phi}_0}{\phi_0} = \frac{\dot{\phi}_0^2}{\phi_0^2} + 3H\,\frac{\dot{\phi}_0}{\phi_0}$$

Generalized Friedmann equations

$$\frac{d\phi(z)}{dz} + (1+z)^2 \frac{c_h}{\phi(z)\,h(z)} = 0$$

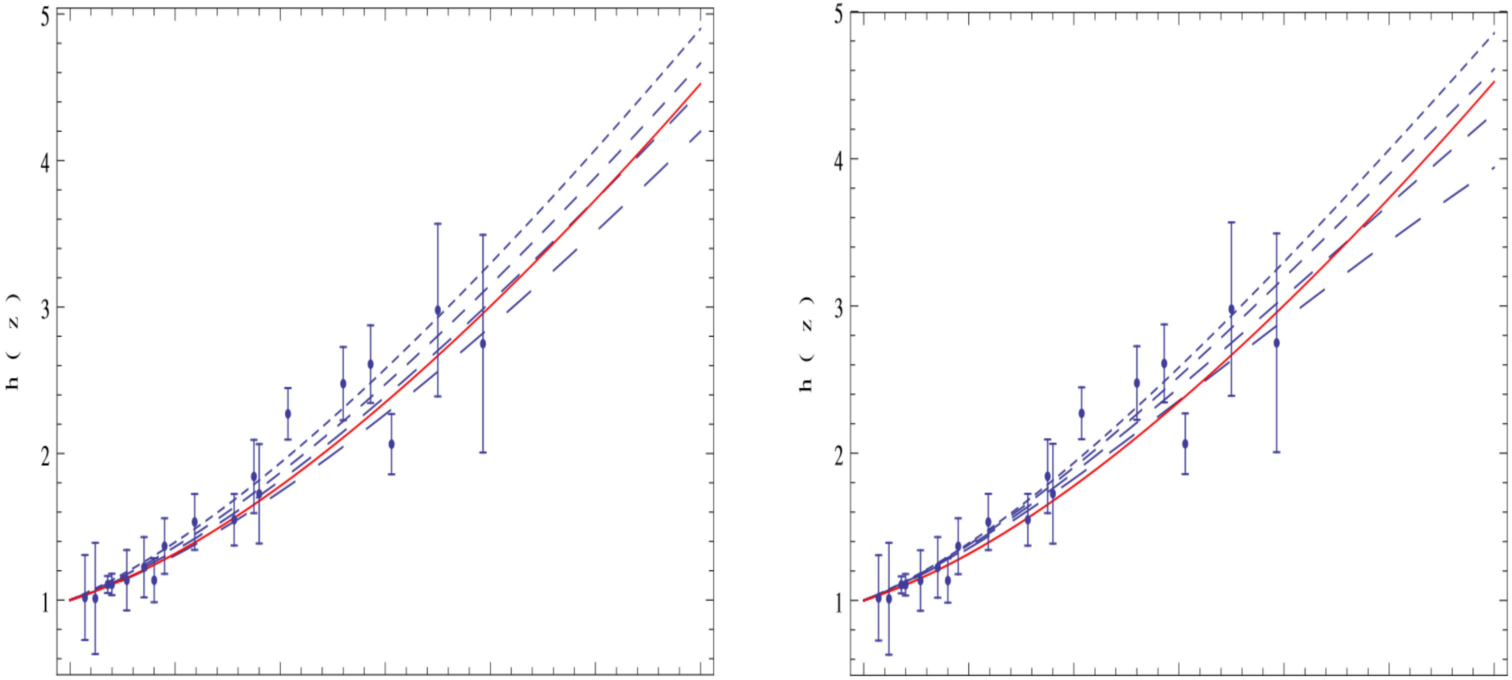
$$\frac{d\tilde{\omega}(z)}{dz} + \frac{u(z)}{(1+z)\,h(z)} = 0$$

$$\frac{du(z)}{dz} - \frac{u(z)}{1+z} - \frac{\lambda\,\phi^2(z)\,\tilde{\omega}(z)}{(1+z)h(z)} = 0$$

$$\frac{dh(z)}{dz} - \frac{3h(z)}{2(1+z)} - \frac{\lambda\,\phi^2(z)\,\tilde{\omega}^2(z) - u^2(z)}{2\,h(z)\,\phi^2(z)}(1+z) + \frac{3\,c_h^2\,(1+z)^5}{2\,\phi^4(z)\,h(z)} + \frac{\phi^2(z)}{8(1+z)h(z)} = 0, \quad (53)$$

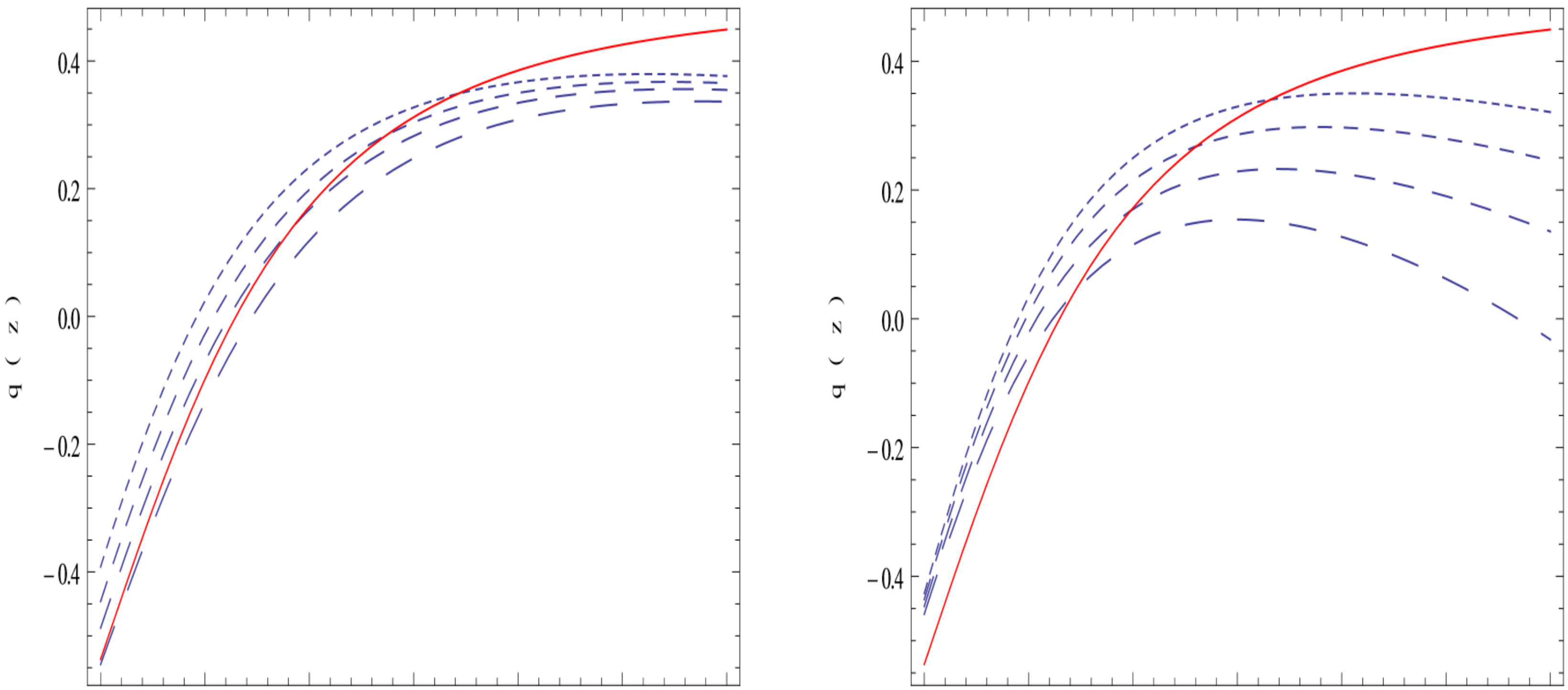
$$h^2(z) - \frac{(1+z)^2}{\phi^2(z)}\left[\lambda\phi^2(z)\tilde{\omega}^2(z) + u^2(z)\right] + \frac{c_h^2\,(1+z)^6}{\phi^4(z)} + \frac{6\,c_h\,h(z)}{\phi^2(z)}(1+z)^3 - \frac{\phi^2(z)}{12} = 0. \quad (54)$$

Generalized Friedmann equations



The dimensionless Hubble function $h(z)$ in ΛCDM (red curve) and in Weyl cosmology as a function of the redshift for initial conditions: $h(0) = 1$, $\phi'(z = 0) = 0.06$ and with different $\phi(z = 0) = 2.67$ (dotted curve), $\phi(z = 0) = 2.75$ (short dashed curve), $\phi(z=0) = 2.81$ (dashed curve) and $\phi(z=0) = 2.89$ (long-dashed curve).

Generalized Friedmann equations



The deceleration parameter $q(z)$ in ΛCDM (red curve) and in Weyl cosmology as a function of the redshift for initial conditions: $h(0) = 1$, $\phi'(z = 0) = 0.06$ and with different $\phi(z = 0) = 2.67$ (dotted curve), $\phi(z = 0) = 2.75$ (short dashed curve), $\phi(z=0) = 2.81$ (dashed curve) and $\phi(z=0) = 2.89$ (long-dashed curve).

Generalized Friedmann equations

Cosmological constraints on the Weyl and gravitational couplings

$$\alpha^2 \omega_3^2(0) \approx 0.22\, H_0^2, \qquad \xi^2 \left[\frac{d\omega_3}{dz}\right]_{z=0}^2 \approx 1.24\, H_0^2$$

H0=H0(α, ξ, ω)

Conclusions

- Weyl geometry may represent the bridge between elementary particle physics, based on the gauge principle, and Einstein gravity
- It allows a natural embedding of the Standard Model *without* any additional degrees of freedom

Weyl conformal geometry alone provides a natural explanation of the present-day cosmological dynamics

Observational consequences could be detected at the level of the Solar System