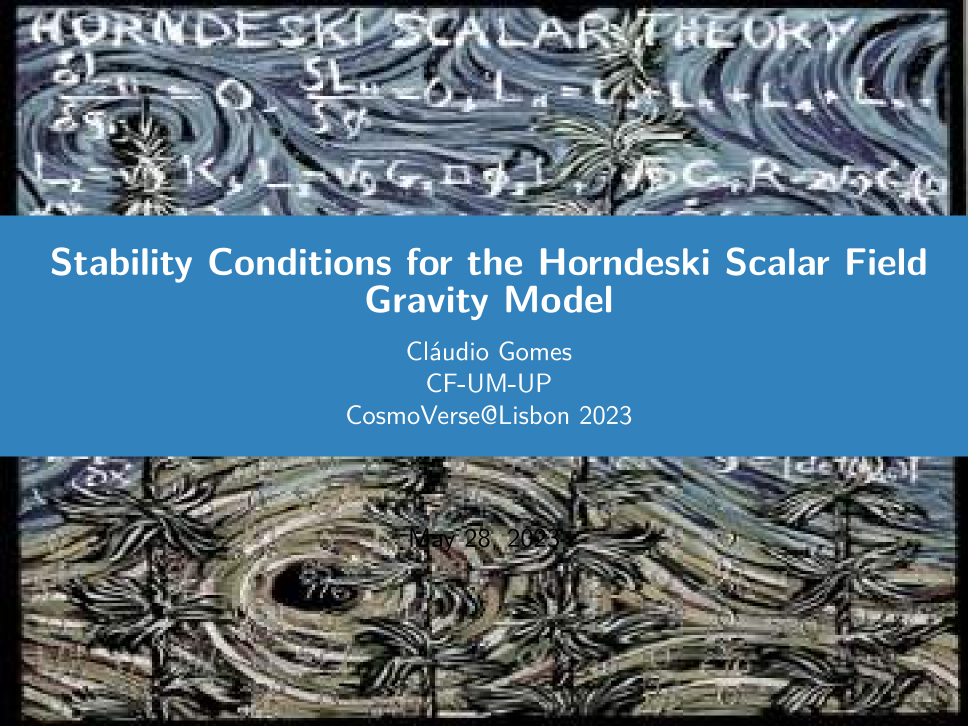

Stability Conditions for the Horndeski Scalar Field Gravity Model

Cláudio Gomes
CF-UM-UP

CosmoVerse@Lisbon 2023

May 28, 2023

0 Outline

1 Outline

1 GR got some serious injuries...

Successes:

- Perihelion precession of Mercury's orbit; deflection of light by the Sun; gravitational redshift of light;
- Incredible match with Solar System constraints;
- "Accurate" black hole shadows;
- Metric Tensor Perturbations at speed of light (LIGO, Virgo collaborations);
- ...

Bruises:

- Singularities;
- Dark Matter and Dark Energy;
- Quantum version (seems incompatible with QM);
- Cosmological Constant;
- ...

1 Treatments

Bandages and some ibuprofen:

- DE is a cosmological constant and DM is cold (beware of colds nowadays!);
- Singularities shall be solves once full quantum gravity is found (sometime, somewhere, someone elses);
- It matches most of the observations (so did Newton's theory for several centuries)
- GR is beautiful, hence leave it that way!

Surgeries:

- Look for theories beyond GR;
- How to "change the gravity" of this situation?
  - Modify the pure gravity sector of Einstein's field equations (e.g. f(R));
  - Modify the matter content sector of Einstein's field equations (e.g. DM, DE);
  - Both (e.g. Non-minimal matter-curvature couplings).
- ...

1 Wedding Proposals (Gravity Theory and Observations)

Several Alternative Theories of Gravity have been proposed in the literature, namely:

- Horndeski/Generalised Galileon (additional scalar field);
- Further properties of spacetime (torsion, non-metricity);
- Non-minimal couplings ($f(R, L)$, $f(R, T)$, $f(R, \mathcal{R}), ...$);
- Loop Quantum Cosmology;
- $p-$forms;
- ...

Our focus here: Horndeski gravity.

2 Outline

2 Greggory Horndeski, the... painter?

- 1974: proposes a ghost-free scalar theory up to second order derivatives.
- 1976: extends the model relying on an Abelian vector field whose action is invariant under $U(1)$ transformations (in flat spacetime = Einstein-Maxwell action).
- His works remain ignored. Switches career to... painter!
- 2014 emerges a different formulation of his theory: Generalised Galileon theories.
- Unaware of his success until a PhD student asked him for a painting for thesis on his gravity model.
- Contains: GR, Brans-Dicke, Quintessence, Dilaton, Chameleon, covariant Galileon...

2 Horndeski and Galileons

Classical Galileon: is a field, π, which obeys to a Galilean symmetry $\pi \to \pi + b_\mu x^\mu + c$ (μ, c constant 4-vector and scalar).

Covariant Galileon: breaks Galileon symmetry, but gives origin to field equations of order non higher than two in the spacetime derivatives, in such way that both classical and quantum pathologies are absent [Deffayet et al, 2011,2013].

Equivalence: Horndeski scalar gravity and Generalised Galileon Gravity are equivalent to each other at least in four dimensions [Kobayash et al, 2011].

After GW170817:

$$S = \int d^4x\sqrt{-g}\left[\sum_{i=2}^{5}\mathcal{L}_i + \mathcal{L}_M\right] \ , \tag{1}$$

where g stands for the metric determinant, $\mathcal{L}_M$ is the matter Lagrangian density and $\mathcal{L}_i$ are the Horndeski Lagrangian densities defined as:

$$\mathcal{L}_2 := G_2(\phi, X) \ , \tag{2}$$

$$\mathcal{L}_3 := G_3(\phi, X)\Box\phi \ , \tag{3}$$

$$\mathcal{L}_4 := -G_4(\phi)R \ , \tag{4}$$

$$\mathcal{L}_5 := G_5^{(0)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \ , \tag{5}$$

where $G_i(\phi, X)$ are arbitrary functions of the scalar field, ϕ, and its kinetic term, $X := \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$, R is the scalar curvature, and $G_{\mu\nu}$ is the Einstein tensor.

3 Outline

3 The Energy Problem in Gravity

Energy: In GR, the gravitational field has no local energy-momentum density. However, for asymptotically flat space-times there is a unique total energy-momentum four vector which is conserved. Still some freedom on the choice: ADM mass, Komar's mass or Bondi mass.

Schoen and Yau, 1979: A lengthy proof of the conjecture that regular space-times with physically reasonable matter would have positive total energy.

Witten, 1981: Elegant demonstration. Stability of supersymmetric solutions of supergravity theories. Ground state of the N=1 SUGRA, Minkowski space is stable both classically and semiclassically.

3 Witten's version of The Positive Energy Theorem

It can be shown that the total energy-momentum tensor for an asymptotically flat space-time is given in terms of the connection by [Nester 1981]:

$$16\pi G V^{\lambda} p_{\lambda} = -\oint_{S} V^{\lambda} \delta^{\sigma\alpha\beta}_{\mu\nu\lambda} g^{\nu\delta} \Delta\Gamma^{\mu}{}_{\delta\beta} \frac{1}{2} dS_{\sigma\alpha} \ , \tag{6}$$

which can be recast, resorting to a two-form $E^{\sigma\alpha} = 2\left(\bar{\epsilon}\Gamma^{\sigma\beta}\nabla_{\beta}\epsilon - \bar{\nabla_{\beta}\epsilon}\Gamma^{\sigma\alpha\beta}\epsilon\right)$, with ϵ being the Dirac spinor, $V^{\mu} = \bar{\epsilon_0}\gamma^{\mu}\epsilon_0$ with γ^{μ} the Dirac's matrices, as:

$$16\pi G V^{\lambda} p_{\lambda} = \int_{\Sigma} \nabla_{\alpha} E^{\sigma\alpha} d\Sigma_{\sigma} \tag{7}$$

3 Extensions of Witten's work

For any supersymmetric theory [Gibbons et al, 1983]:

$$\hat{\nabla}_\alpha E^{\sigma\alpha} = 2G^\sigma_\alpha \bar{\epsilon}^i \gamma^\alpha \epsilon^i + 4\overline{\hat{\nabla}_\alpha \epsilon^i} \Gamma^{\sigma\alpha\beta} \hat{\nabla}_\beta \epsilon^i + \overline{\delta\chi^a} \gamma^\sigma \delta\chi^a \ , \tag{8}$$

In General Relativity

$G_{\mu\nu} \propto T_{\mu\nu}$, the positiveness of the integrand is ensured provided the energy-momentum tensor of matter fields satisfies the dominant energy condition, $\bar{\epsilon}^i_0 \gamma^\alpha \epsilon^i_0$ is non-spacelike, and the Witten condition $\gamma^\lambda \hat{\nabla}_\lambda \epsilon^i = 0$ is chosen.

For nonsupersymmetric theories [Boucher,1984]:

$$\hat{\nabla}_\alpha \epsilon^i := \nabla_\alpha \epsilon^i + \frac{i}{2} \hat{k} \gamma_\alpha f_1^{ij} \epsilon^j \ , \qquad \delta\chi^a = i\gamma^\lambda \nabla_\lambda \phi f_2^{ai} \epsilon^i + f_3^{ai} \epsilon^i \ . \tag{9}$$

For scalar field nonminimally coupled to gravity [Bertolami, 1987].
For noncommutative scalar field coupled to gravity [Bertolami, Zarro, 2008].

4 Outline

4 Results: [C. Gomes, O. Bertolami, JCAP 04, 008 (2022)]

We found that: $G_3 = G_3(\phi)$ and:

$$\begin{cases} G_{2X}(\phi, X)\delta^{ij} = \left(4G_{3\phi}(\phi) - 6G_{4\phi\phi}(\phi)\right)\delta^{ij} \\ 24\left(\hat{\kappa}(\phi)f_1(\phi, X)^{ij}\right)^2 \delta^{\sigma}{}_{\alpha} = f_3(\phi, X)^{ai} f_3(\phi, X)^{aj}\delta^{\sigma}{}_{\alpha} + \\ \quad + 2\hat{\kappa}(\phi)\left(G_2(\phi, X)\delta^{\sigma}_{\alpha} + 2G_{4\phi}(\phi)\Box\phi\delta^{\sigma}_{\alpha} - 2G_{4\phi}(\phi)\nabla^{\sigma}\nabla_{\alpha}\phi\right)\delta^{ij} \\ 4\left(\hat{\kappa}(\phi)f_1(\phi, X)^{ij}\right)_{\phi} = f_2(\phi, X)^{ai} f_3(\phi, X)^{aj} + f_2(\phi, X)^{aj} f_3(\phi, X)^{ai} \\ f_2(\phi, X)^{ai} f_2(\phi, X)^{aj} = 2\hat{\kappa}(\phi)\left(-G_{3\phi}(\phi) + 2G_{4\phi\phi}(\phi)\right)\delta^{ij} \end{cases}, \quad (10)$$

Together with the boundary condition (by inspection of the N=1 SUGRA):

$$f_1^{ij}(\phi_0) = \sqrt{\frac{G_2(\phi_0)}{6G_4(\phi_0)}}\delta^{ij} \ . \quad (11)$$

Zero energy states are found to be stable.

4 Other Stability Criteria

Attractive Gravity

$$\hat{\kappa} > 0 \ , \tag{12}$$

which is also satisfied by inspecting both numerator and denominator of the speed of sound for avoiding of ghost and gradient instabilities, i.e., $G_4(\phi) > 0$.

Dolgov-Kawasacki instabilities The trace of the field equations can allow for a dynamical equation for R, hence the associated "squared mass" can be non-positive. For the viable models of Horndeski scalar gravity an algebraic equation if found, hence no DK instability is expected.

$$R = -\frac{1}{2G_4}\left(T + \hat{T}\right) \ . \tag{13}$$

4 Cosmological implications

Inflation In the absence of matter fields, Horndeski viable models should behave as:

$$H^2 = \frac{8\pi G}{3} V_{eff}(\phi) \,, \tag{14}$$

with $V_{eff} = G_2(\phi)/G_4(\phi)$ (in slow-roll).

Cosmological Constant Upon a suitable identification with N=1 SUGRA, the cosmological constant arises from:

$$\Lambda = \frac{G_2(\phi)}{2G_4(\phi)} \,, \tag{15}$$

together with the solutions $f_1(\phi) = 2\sqrt{\frac{\Lambda}{3}} G_4(\phi)$, $G_3(\phi) = \frac{3}{2} G_{4\phi\phi}(\phi)$ and $f_2(\phi) = \frac{G_{3\phi}(\phi)}{3G_4(\phi)}$.

5 Outline

5 Final Remarks and Work in Progress

- Horndeski scalar gravity model as an alternative theory of gravity
- Witten's theorem applied to Horndeski gravity
- Stability criteria

Work in Progress [to appear soon]: Analysis of Degenerate Higher-Order Scalar-Tensor theories (DHOST).

どうしてDHOST?

Extension of Horndeski (and Beyond Horndeski) theories up to cubic terms, although not involving ghost instabilities.

Thank you for your attention!